NON-LINE-OF-SIGHT ELECTRO-OPTIC LASER COMMUNICATIONS IN THE MIDDLE ULTRAVIOLET.

Dennis Michael Junge

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NON-LINE-OF-SIGHT ELECTRO-OPTIC LASER COMMUNICATIONS IN THE MIDDLE ULTRAVIOLET

by

Dennis Michael Junge

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Thesis Advisor: W.M. Tolles

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(20. ABSTRACT Continued)

of such a system.

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Non-Line-of-Sight Electro-Optic Laser Communications in the Middle Ultraviolet

by

Dennis Michael Junge
Lieutenant, United States Navy

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ABSTRACT

A Monte Carlo computer simulation was developed to model hypothesized electro-optic laser communication systems operating in the middle ultraviolet region of the spectrum called the solar blind. By assuming various source, propagation, and detector characteristics as well as certain performance parameters it is possible to predict the effective ranges and operating characteristics of such a system.

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I. INTRODUCTION

A. HISTORY

Historically optical communications systems were developed over line-of-sight paths due to the propagation characteristics of light. In the United States Navy line-of-sight communications became somewhat neglected with the advent of uhf radio technology. For years the only optical communication systems in the navy were the 12 inch (and for a time 24 inch) hand modulated signal lamps, yardarm blinkers, flag hoist, semaphore, and NANCY (a hand modulated infrared system). Some experimentation was conducted in the late 1940's and 1950's using cesium lamps operating in the near infrared (ir) and non-line-of-sight or over-the-horizon (OTH) communication was demonstrated. Interest waned, however, and progress stood still.

Technology has not stood still and today virtually any radio signal can be monitored and the transmitter localized using a direction finding receiver. While communication systems must be reliable and secure, sometimes it is important that they be covert (only detectable by friendly forces). Thus armed with a complete set of radio communication equipment, the unit commander nevertheless may find his forces in EMCON, talking to each other via flashing light.

B. VISIBLE LIGHT

In the 1960's several attempts were again made to demonstrate the feasibility of OTH and over-terrestrial-obstacles communication using atmospheric scattering [1-3]. These efforts utilized visible search lights, flash lamps, and lasers, and communication at night was successfully demonstrated over distances up to fifty kilometers.

Visible radiation is highly attractive because of the low extinction coefficients (depending on weather conditions). Unfortunately it is seriously degraded during daylight hours due to the background radiation from the sun which tends to interfere with detection.

C. INFRARED LIGHT

Recently, efforts have been made to study extended line-of-sight communications utilizing lasers operating in the near ir. One such effort utilized a 1.06 μ laser and employed remotely piloted vehicles (RPVs). Communications at this wavelength appeared quite promising and ranges from 25 to 150 nautical miles have been predicted [4].

In this region of the spectrum, however, there is noticeable background noise due to scattering of incident solar radiation and to emission by atmospheric particles heated by incident radiation. It should be noted that the daylight background visible radiance is due primarily to scattered radiation and night radiance is caused mainly by scattered radiation and moon light. In the near ir daylight

background radiance is due mainly to atmospheric emission, while at night aurora and afterglow are the predominate sources.

D. ULTRAVIOLET LIGHT

There is a portion of the spectrum, however, that has been studied little and may offer much. This is the middle ultraviolet (uv). Atmospheric absorption in the .2 to .3 μ region is caused primarily by ozone [5-6]. The relatively dense layer of ozone (at approximately 22 km) prevents virtually all of the radiation in the 220 to 280 nm region from reaching the surface of the earth. Thus the limiting noise in an electro-optical communication system operating in the middle uv would be self-noise rather than atmospheric noise. Another critical feature to be considered is the attenuation properties of the intervening atmosphere. In the near uv (wavelengths between .3 and .4 μ) the primary attenuation mechanism at low altitude is scattering. This can be divided into Rayleigh scattering and particulate scattering by atmospheric aerosols. Rayleigh scattering, inversely proportional to the fourth power of wavelength, increases quite rapidly as the wavelengths become shorter. In practice, it has been difficult to determine the particulate scattering in the near uv. For clear sky conditions, however, a reasonable estimate of attenuation can be made by assuming that the entire attenuation is due to Rayleigh scattering [2].

In the middle uv particulate scattering is very significant. At high altitudes the large ozone concentration

gives rise to very large absorption coefficients. The concentration of ozone generally decreases as the altitude is decreased. Yet even at sea level the concentration is still high enough to have a significant effect. These propagation characteristics indicate that only short range communication systems operating in the middle uv are likely to be useful. Because of the inherent scattering characteristics of middle uv radiation, along with the minimal background problem in the solar blind region, short range non-line-of-sight communications utilizing broad band light sources have been developed [7-8].

Recently a number of reasonably efficient uv lasers have been developed with energies of .1 to one joule per pulse being reported [9-10]. These represent a new generation of radiation sources to be considered for use in the solar blind. A number of detectors and filters have been available for some time. It was these considerations that led to this study.

E. PURPOSE

The purpose of this effort is to develop an analytical tool for estimating the ranges for which optical communication is possible in the middle uv.

II. BACKGROUND

A. INTRODUCTION

A typical communication system consists of a transmitter, a receiver, and the intervening space between them. The characteristics of the transmitter and the receiver are highly dependent on each other and on the properties of the propagating medium. Each of the components may, however, be treated as a distinct and relatively separate entity. The radiation source, or transmitter, must be suitably modulated to carry information and must have sufficient power to overcome the attenuating mechanisms in the medium in which it operates. The characteristics of the propagating medium are not well defined and in general must be approximated. The detector, or receiver, must have sufficient sensitivity to detect and demodulate the attenuated signal that is embedded in background noise. Each of these components is present in an electro-optical communication system and is discussed below. It will be seen that 1) advances in sources dictate a new look at the problem; 2) uncertainties in propagation characteristics of the middle uv requires careful experimentation to gather suitable information such that a proposed system may be modeled; and that 3) certain aspects of detectors, such as filters, critically determine the performance parameters of such a system.

B. SOURCES

Ultraviolet sources suitable for communication purposes include omnidirectional broad band lamps such as xenon or mercury flash lamps, and lasers. Experiments utilizing omnidirectional sources have been carried out for several years at the Aeronautical Research Associates of Princeton (ARAP), Princeton, New Jersey [7-8]. Utilizing a pulsed source at 4810 pps and an average power level of ten to thirty watts, voice communication was demonstrated over a distance of several hundred meters. With suitable modifications, it is estimated that ranges of up to three kilometers may be achieved with the current system. With such a source, the conversion efficiency from input power to output power is about 0.1 percent.

New uv laser sources reported in the last year or so appear to have demonstrated orders of magnitude improvement in efficiency. Utilizing rare gas-halide eximer species, uv output with working efficiencies of up to ten percent are conceivable. Practical working models are already commercially available. Wavelengths in the range of 200 to 400 nm have been reported, indicating that various wavelengths in the middle uv may be attained with these sources. The power output of these devices typically runs from 0.1 joules upwards, depending on the size of the device. The high degree of collimation of the output suggests that the geometric patterns for communication will be highly directional. This is one aspect of the problem which is quite different from that observed for omnidirectional sources.

C. PROPAGATION

The absorption coefficient for middle uv radiation varies by orders of magnitude as a function of wavelength. Scattering by particulates at these wavelengths is more severe than in the visible region of the spectrum. The combined absorption and scattering cross-sections give rise to extinction coefficients which have been measured to be in the range of 0.1 to five per kilometer [11]. Further, variations of an order of magnitude in extinction coefficient over a spectral range of less than fifty nanometers indicates extreme sensitivity to wavelength, due largely to the absorption bands of species such as ozone which appear in this region.

At wavelengths less than 280 nm, the absorption coefficient of atmospheric ozone is sufficiently intense as to eliminate the large majority of solar radiation from the earth's surface [12]. This is largely due to an ozone concentration centered at 20 to 25 km above the surface of the earth. At sea level, the ozone concentration is typically an order of magnitude less than at the higher altitudes. This gives rise to reduced solar background levels without the high extinction coefficients found at higher altitudes. Such behavior suggests that communication in the middle uv may be feasible for relatively short distances at sea level, with a minimum of interference from solar background radiation.

Further major considerations for middle uv propagation are the effects due to multiple scattering. Although the primary light beam may be attenuated by an extinction coefficient which is readily measured, the scattered radiation, especially after multiple scattering, may contribute considerable intensity to the radiation density at an observer. Shettle and Green [12] present calculations for solar flux as a function of the sun's angle with respect to the zenith. Particularly at low angles of incidence, the diffuse radiation is found to be twenty to thirty orders of magnitude more intense than that due to the direct solar radiation. Such figures clearly emphasize the importance of properly treating the effects of multiple scattering when taking into account the effects of particulates in the atmosphere.

In general multiple scattering calculations are cumbersome to handle and yield limited information. Typically these calculations are done on a computer using either an analytical [13] or a Monte Carlo [14-15] appraoch. The Monte Carlo calculations are relatively easier to model, but utilize a significant amount of computer time and only approximate information is obtained. Some generally useful results have been presented by Bucher [14-15] again indicating orders of magnitude greater radiation densities from the multiple scattering effects than from transmission of direct radiation. The available results from such calculations do not readily answer questions such as 1) what is the

scattering intensity for non-line-of-sight communication given an obstacle with an assumed angle between transmitter-obstacle-receiver; and 2) what is the actual radiation density as a function of field of view and/or position of an observer located off from the axis of propagation at a given distance from a collimated source such as a laser.

D. DETECTION/DEMODULATION

The information available from a pulsed signal depends on the pulse width, repetition rate, and signal-to-noise ratio. Considerations such as these are included in papers by Kennedy [16-17] in which communication through optical scattering channels is discussed. Considerations leading to a suitable signal-to-noise ratio analysis are also presented by Yarif [18] for electro-optical systems. Whether the information is pulsed or continuous wave greatly affects the reception characteristics of the signal. The background light levels coming through the filters utilized before the detector critically determine the signal-to-noise ratio. The characteristics of the filter and detector thus are crucial in determining the information available in a communication system.

III. STATEMENT OF THE PROBLEM

One of the primary constraints on the development and deployment of any new system in the United States Navy is the limited fiscal resources available for such efforts. With the advent of new technology it is nevertheless impossible to utilize the technology to build a new system without first having a mission or demonstrated need for that system and second having a well-defined degree of confidence in the ability of that system to meet that need.

An electro-optical communication system utilizing a laser operating in the middle uv is such a system. (It requires little or no imagination to visualize the advantages of rapid short range, possibly non-line-of-sight, optical communications over the methods presently employed by the Fleet.) While it is beyond the scope of this project to develop such a system, it is within the scope of this effort to formulate a model of such a system capable of predicting the various parameters of that system.

The need for modeling is further enhanced by the limited amount of information available concerning the middle uv. By utilizing available information and by characterizing source, propagation, filter and detector characteristics, it is possible to formulate a model that will yield order of magnitude results for any proposed system. In the process of modeling and analysis the need for critical experiments

which would allow the modeling efforts to proceed with a greater degree of accuracy is sure to be revealed.

IV. LASER SOURCES

The past two or three years have seen a rapid development of lasers operating in the ultraviolet. The most notable recent advances have occurred with the rare gas-halide lasers in which an excited state eximer is formed. With species such as XeF*, the ground state is not an associated molecule, thus depopulation of the lower level is not a barrier to population inversion. Further, such rare gas-halide lasers have an inherently high quantum efficiency. Efficiencies relative to the energy deposited by an excitation electron beam are as high as ten percent, while overall working efficiencies of up to one percent have been reported [9,10,19]. With higher overall efficiencies anticipated, such lasers represent an attractive source of radiation for middle uv applications.

The observed wavelengths of several rare gas-halide lasers are presented in Table I. It should be noted that the laser output from these sources may be Raman shifted with reasonable high conversion efficiencies [20]. By passing the output from a KrF laser through ten atmospheres of hydrogen, conversion to the first stokes wavelength of 25 percent and to the second stokes wavelength of ten percent was observed [20]. At eighty atmospheres pressure, more than fifty percent conversion efficiency to the first stokes wavelength has been observed.

TABLE I

Characteristic Wavelengths of Several Rare Gas-Halide Lasers Along with Some Stokes Shifted Wavelengths Observed with Hydrogen Gas

Eximer	Wavelength (nm)	First Stokes Wavelength (nm)	Second Stokes Wavelength (nm)
KrF	249	279	318
XeF	351		
XeCl	308		
XeBr	282	322	

The practical performance of rare gas-halide lasers is illustrated by the commercial availability of a model which will produce 0.1 joule pulses at 15 pps. One table top model is reported to produce 1.5 joule pulses with a 125 nanosecond pulse width. The development of these lasers is in its infancy. Sources with higher repetition rates and greater reliability are to be expected.

Due to the rapidly expanding availability of middle uv lasers sources, no single presently available source is assumed, but rather the effort in the succeeding sections considers the possibility of utilizing a middle uv source at representative wavelengths and with an output power consisting of millijoule to joule pulses. Thus the behavior of any unforseen laser or coherent beam may be predicted.

V. PROPAGATION PROPERTIES OF THE ATMOSPHERE AND THE MULTIPLE SCATTERING PROBLEM

A. INTRODUCTION

A laser signal propagating through the atmosphere is subjected to various attenuation mechanisms. The time spread of a pulse can be broadened due to scattering, while the amplitude can be decreased due to absorption and scattering by atmospheric constituents. The inherent directionality of a laser source indicates that the position of the detector relative to the source should also affect the reception of the signal. Since a large fraction of the total extinction process is due to scattering, multiple scattering should be an important consideration affecting the propagation characteristics for distances greater than one or two extinction lengths.

A complete description of all of the effects of the atmosphere including inhomogeneities (clouds, terrestrial objects, surface effects, etc.) is beyond the scope of this paper. A relatively concise treatment of the atmosphere and the multiple scattering problem entails characterizing the atmosphere, simulating atmospheric effects, and tabulating the results of the simulation.

B. CHARACTERISTICS OF THE ATMOSPHERE

1. Attenuation

In order to characterize the propagation of optical information in the atmosphere it was first necesary to

determine the extinction, absorption, and scattering coefficients. Table II was constructed based primarily on the information reported by Dunkelman [21]. The Rayleigh scattering coefficients were adapted from Penndorf [22].

For the purpose of this investigation the following definitions and relationships apply:

λ is the wavelength

K_A is the absorption coefficient

K_{SR} is the Rayleigh scattering coefficient

K_{SP} is the particulate scattering coefficient

K_S is the total scattering coefficient

K is the total extinction coefficient

$K_S = K_{SR} + K_{SP}$

$K = K_S + K_A$

R_S is the ratio of K_S to K or (K_S/K)

R_P is the ratio of K_{SP} to K_S or (K_{SP}/K_S)

From Table II it can be seen that K_A, K_{SR}, and K tend to decrease as wavelength increases in the middle uv. The relationship of K_{SP} and consequently K_S, R_S, and R_P to wavelength is not so easily described.

2. Angular Distribution of Scattered Photons

Complete characterization of the propagation of optical information required the angular distribution or phase function, $P(\theta)$, of the scattered photons for the wavelengths of interest. Figure 1 [23] was the primary source for phase function information.

TABLE II

Attenuation Coefficients and Characteristic Ratios for Middle uv Radiation

λ (nm)	K_A (km^{-1})	K_{SR} (km^{-1})	K_{SP} (km^{-1})	K_S (km^{-1})	K (km^{-1})	R_S (K_S/K)	R_P (K_{SP}/K_S)
230	2.58-3.5	.4934	.03-2.9	.52-3.4	3.1-6.0	.17-.57	.06-.85
240	1.43-2.8	.4061	.26-3.2	.67-3.6	2.1-5.0	.32-.72	.39-.89
250	.68-2.6	.3382	.18-2.4	.52-2.7	1.2-3.4	.43-.79	.35-.89
260	.43-2.4	.2842	.29-2.2	.57-2.5	1.0-2.9	.57-.86	.51-.88
270	.23-1.6	.2404	.33-2.1	.57-2.3	.80-2.6	.71-.88	.58-.91
280	.13- .89	.2055	.065-1.9	.27-2.1	.40-2.2	.68-.95	.24-.90
290	.046- .32	.1765	.078-1.9	.25-2.1	.30-2.1	.83-1.00	.31-.90
300	.012- .081	.1525	.056-1.7	.21-1.9	.22-1.9	.95-1.00	.27-.89
310	.003- .021	.1326	.044-1.6	.17-1.7	.18-1.8	.94	.26-.94

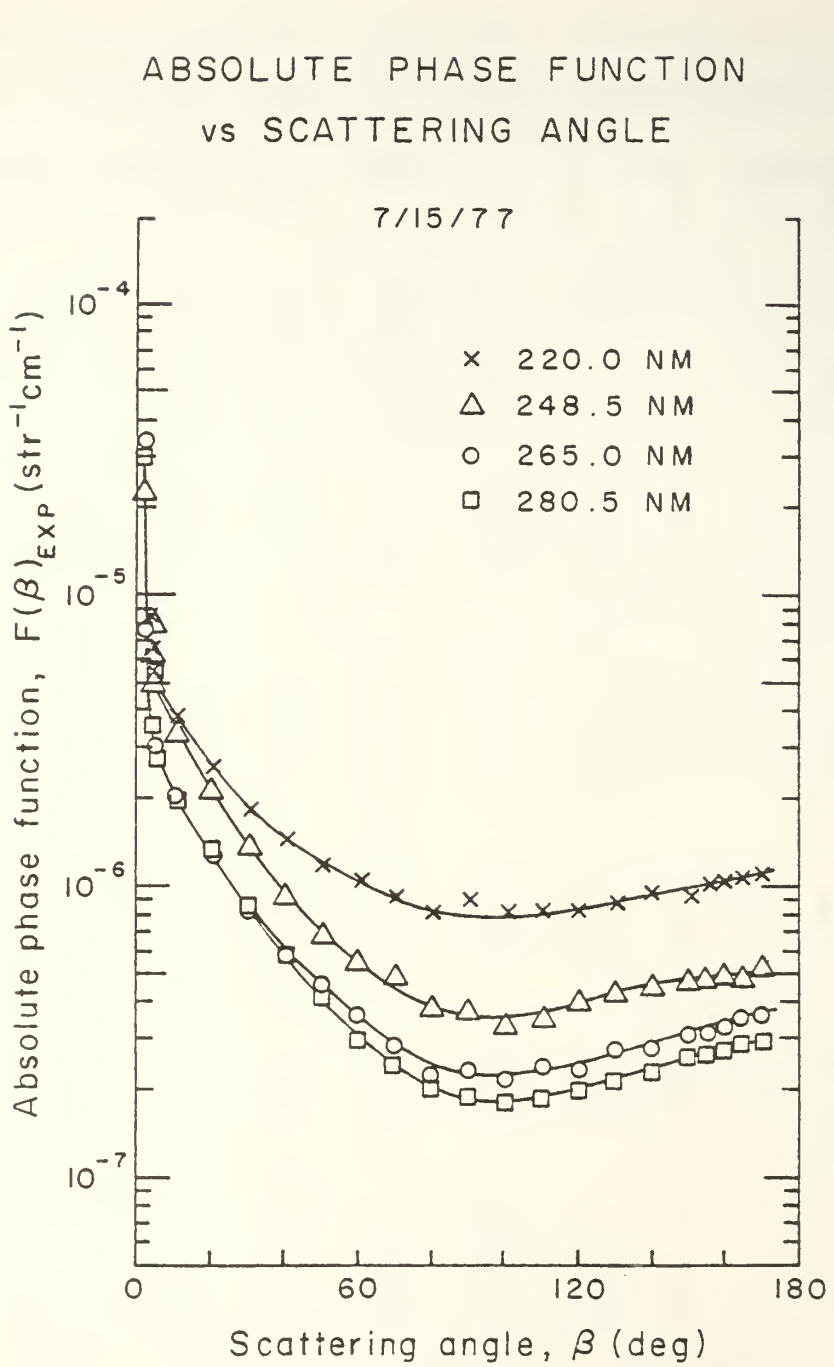

FIGURE 1. Absolute Phase Function vs. Scattering Angle [23]

In order to mathematically model this information, the Henyey-Greenstein function, Eq. 1, and some of its modifications were studied. The Neer-Sandri function [7], Eq. 2, was a logical choice; however, the function used by Zachor [24], Eq. 3, was chosen for simplicity in modifying the author's initial phase function to properly account for backscattering.

$$P(\theta,g) = \frac{1-g^2}{4\pi}\left[\frac{1}{(1+g^2-2g\cos\theta)^{3/2}}\right] \tag{1}$$

$$P(\theta,g) = \frac{1-g^2}{4\pi}\left[\frac{1}{(1+g^2-2g\cos\theta)^{3/2}} + \frac{g(3\cos^2\theta-1)}{2|\cos\theta_0|(1+g^2-2g|\cos\theta_0|)^{5/2}}\right] \tag{2}$$

where $|\cos\theta_0| = 1/7$

$$P(\theta,g) = \frac{1-g^2}{4\pi}\left[\frac{1}{(1+g^2-2g\cos\theta)^{3/2}} + \frac{f\ 0.5(3\cos^2\theta-1)}{(1+g^2)^{3/2}}\right] \tag{3}$$

Fitting the data reported by Neer [23] with Eq. 3 resulted in Figures 2 and 3. These figures represent the absolute and normalized phase functions, while Table III lists the coefficients, utilized for this simulation.

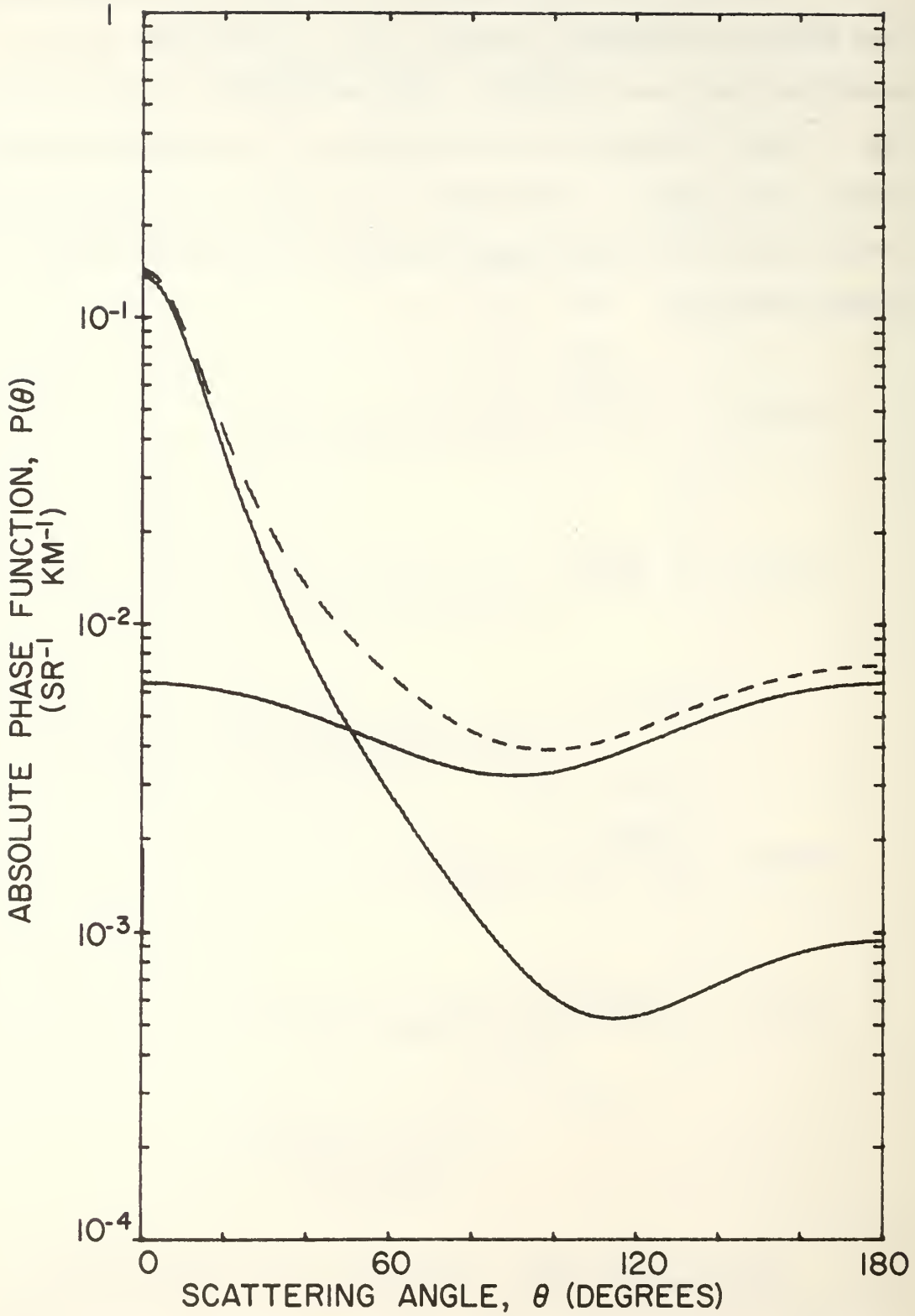

FIGURE 2. Absolute Phase Function vs. Scattering Angle Utilizing a Modified Henyey-Greenstein Function to Approximate the Data by Neer, et al. [23]

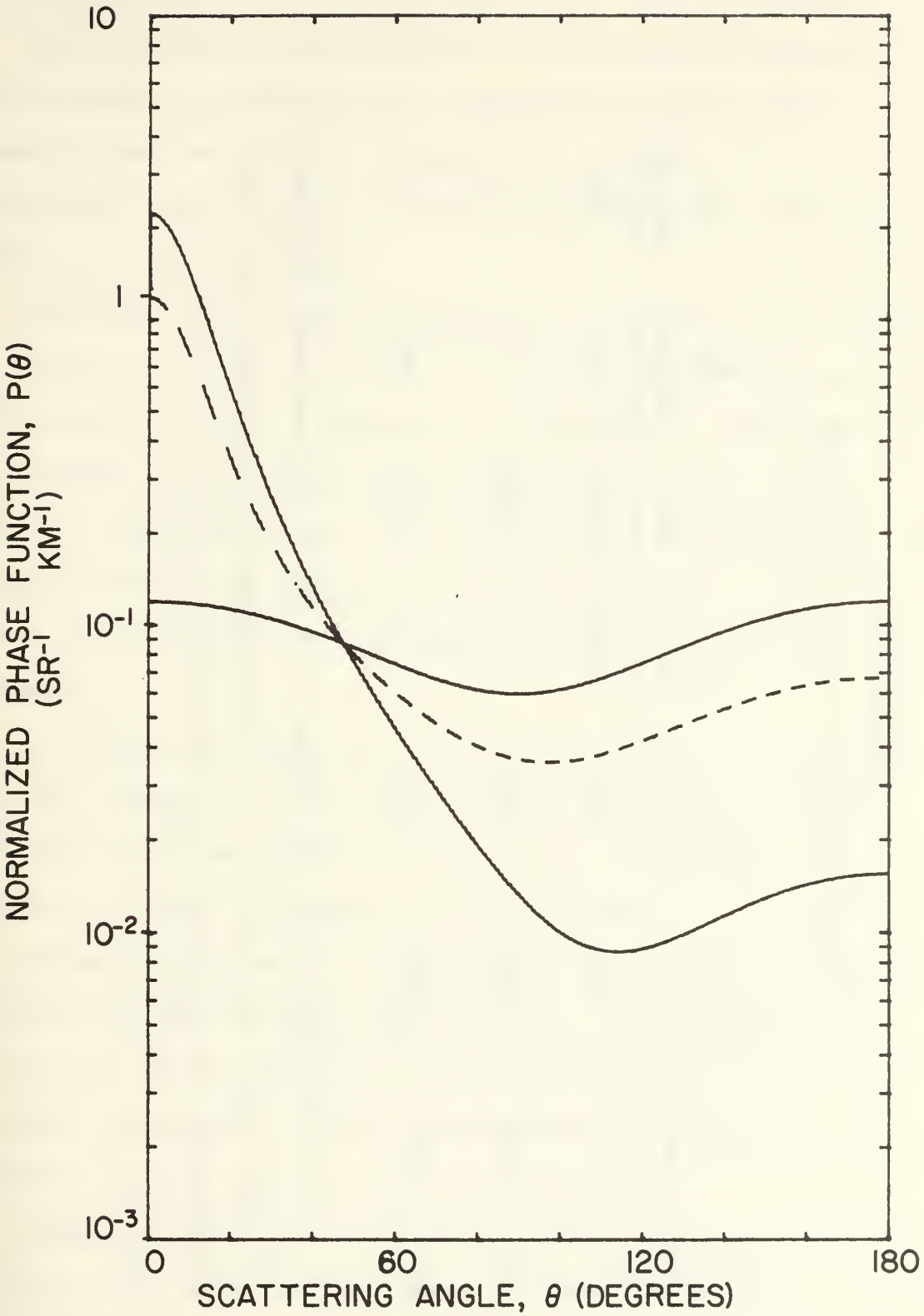

FIGURE 3. Normalized Phase Function vs. Scattering Angle Utilizing a Modified Henyey-Greenstein Function to Approximate the Data by Neer, et al. [23]

TABLE III

Coefficients Used in the Simulation

λ (nm)	K_A (km^{-1})	K_{SR} (km^{-1})	K_{SP} (km^{-1})	K_S (km^{-1})	K (km^{-1})	R_S (K_S/K)	R_P (K_{SP}/K_S)
250	.9372	.3382	.7187	1.0569	1.9941	.53	.68
280	.2185	.2055	.6165	.8220	1.0405	.79	.75
300	.0289	.1525	.5407	.6932	.7221	.96	.78

For all three cases the values of g and f utilized were 0.75 and 0.5, respectively. The values for 250 nm appear to be optimistic while the values for 300 nm appear to be somewhat pessimistic.

C. DESCRIPTION OF THE CHARACTERISTICS OF THE SIMULATION

The purpose of this simulation was to provide approximate results for the multiple scattering problem. By assuming various source, propagation, and detector characteristics, a user could in principle predict the effectiveness of a proposed system.

The Monte Carlo method described in Appendix A was employed to generate information detailing characteristic patterns of photon scattering as a function of wavelength, position of an observer, pulse spreading, and field of view of the detector being employed. The observer may be located at any arbitrary angle off from the axis of propagation and at any arbitrary distance from the source (measured in extinction lengths).

Of primary interest were the characteristic patterns of multiply scattered radiation. The simulation provided for obtaining diagrams of photon flux at a given position as viewed by either the source or the receiver. In addition, the patterns received by the detector were capable of being limited by field of view restrictions. By measuring the photon flux at a given position, contours of equal photon flux were generated. These contours provided a simple means of visualizing the radiation patterns.

Detector field of view considerations indicate the field of view required to receive a given threshold of photons at a fixed position as well as the maximum range at a given angle that a threshold number of photons can be

detected for a fixed field of view. Finally, pulse spreading information due to the time delay caused by multiple scattering was also available.

D. CHARACTERISTIC PATTERNS OF MULTIPLY SCATTERED RADIATION

Figures 4 and 5 represent the pattern of photons received by a 180 degree field of view detector located zero to nine and 36 to 45 degrees off from the axis of propagation respectively at one extinction length from a 300 nm laser source. As the detector approached the axis of propagation more photons were received. The photons along the phi prime equal to zero axis represent single scattered radiation. Many of the photons received were, however, multiply scattered.

Figures 4 and 5 are similar to Figures 6 and 7 only in the latter figures the receiver is located five extinction lengths from the source. The percentages of multiply scattered photons increases greatly as the receiver is moved either off from the axis of propagation or away from the source. At five extinction lengths, virtually none of the radiation received is single scattered. This is a strong indication of the hazards of single scattering models in the middle uv.

Figures 8, 9, and 10 represent the contours of equal photon flux for 250, 280, and 300 nm wavelengths. The label for each contour is

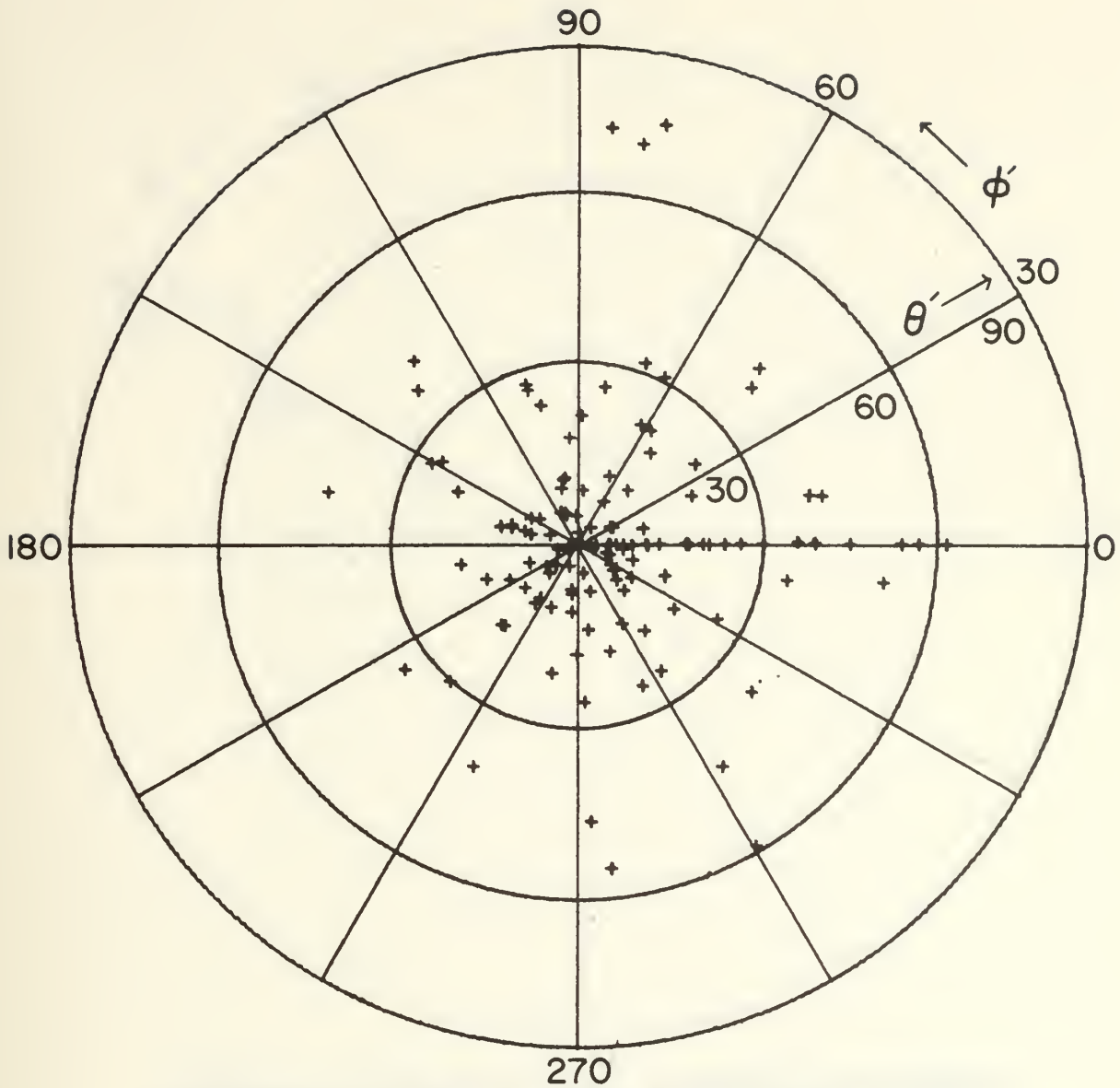

FIGURE 4. Photon Scattering Diagram (θ' vs φ') as Viewed by a Receiver Located θ Degrees from the Axis of Propagation.

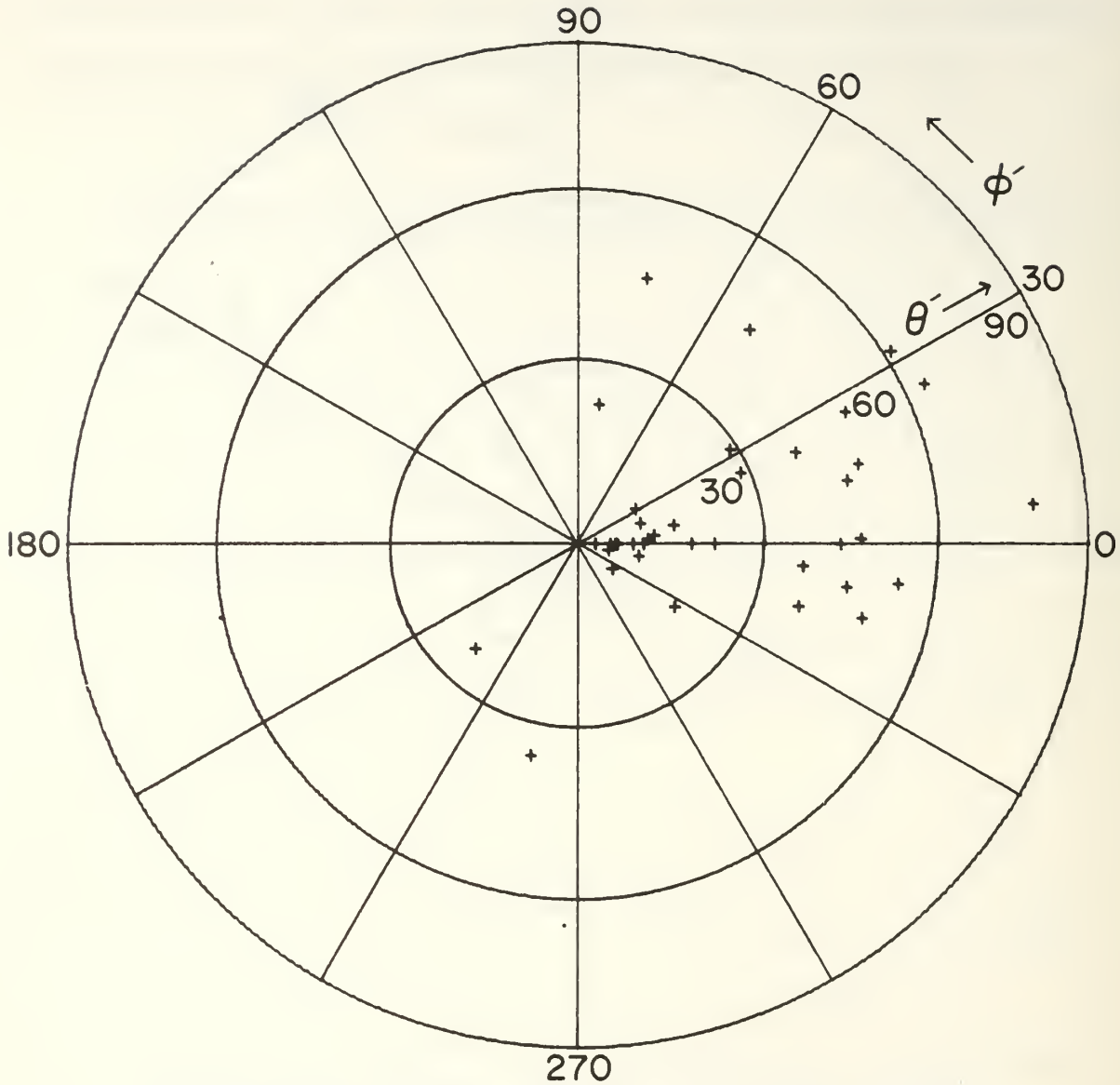

FIGURE 5. Photon Scattering Diagram (θ' vs φ') as Viewed by a Receiver Located θ Degrees from the Axis of Propagation.

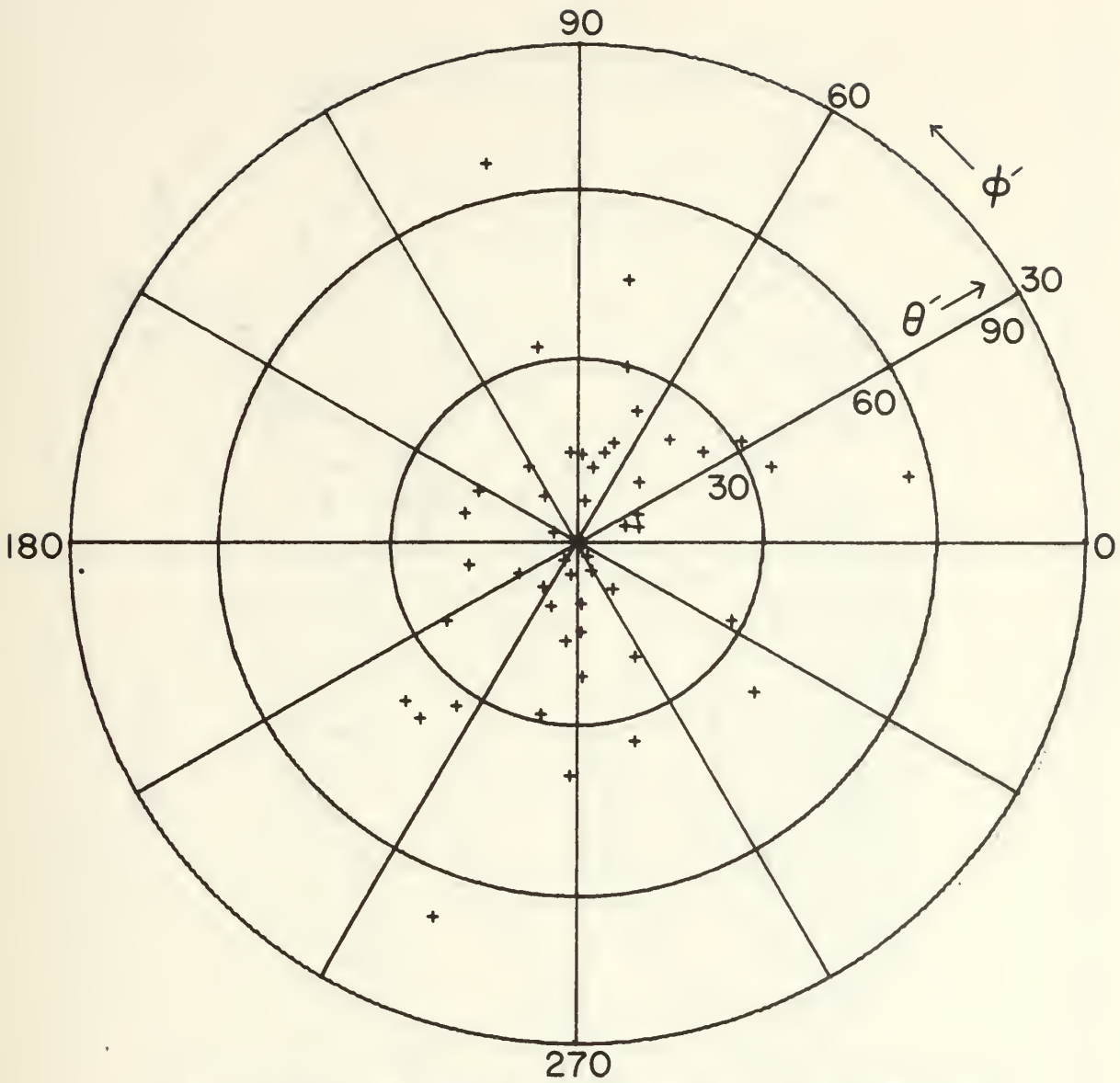

FIGURE 6. Photon Scattering Diagram (θ' vs ϕ') as Viewed by a Receiver Located θ Degrees from the Axis of Propagation

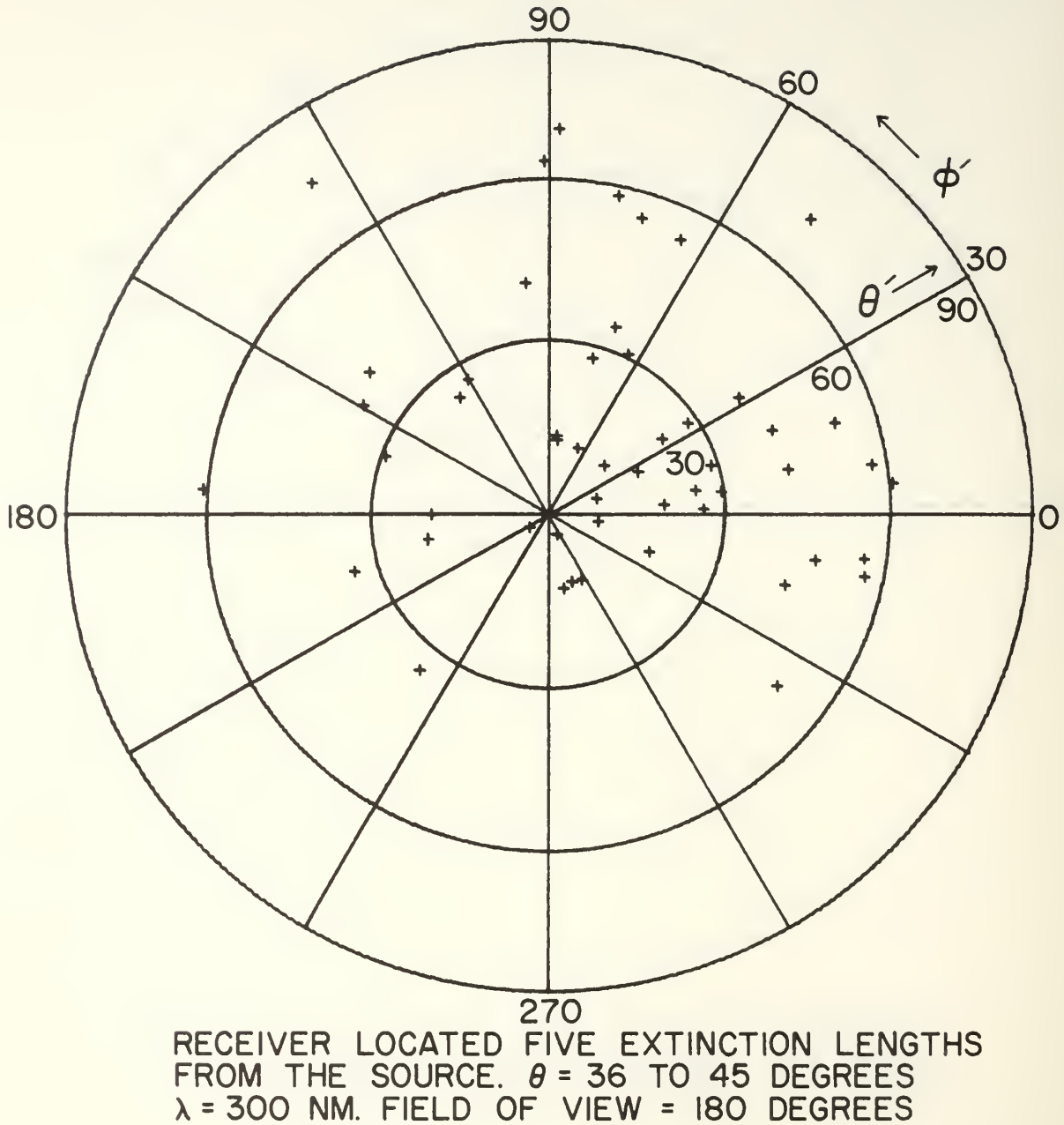

FIGURE 7. Photon Scattering Diagram (θ' vs φ') as Viewed by a Receiver Located θ Degrees from the Axis of Propagation

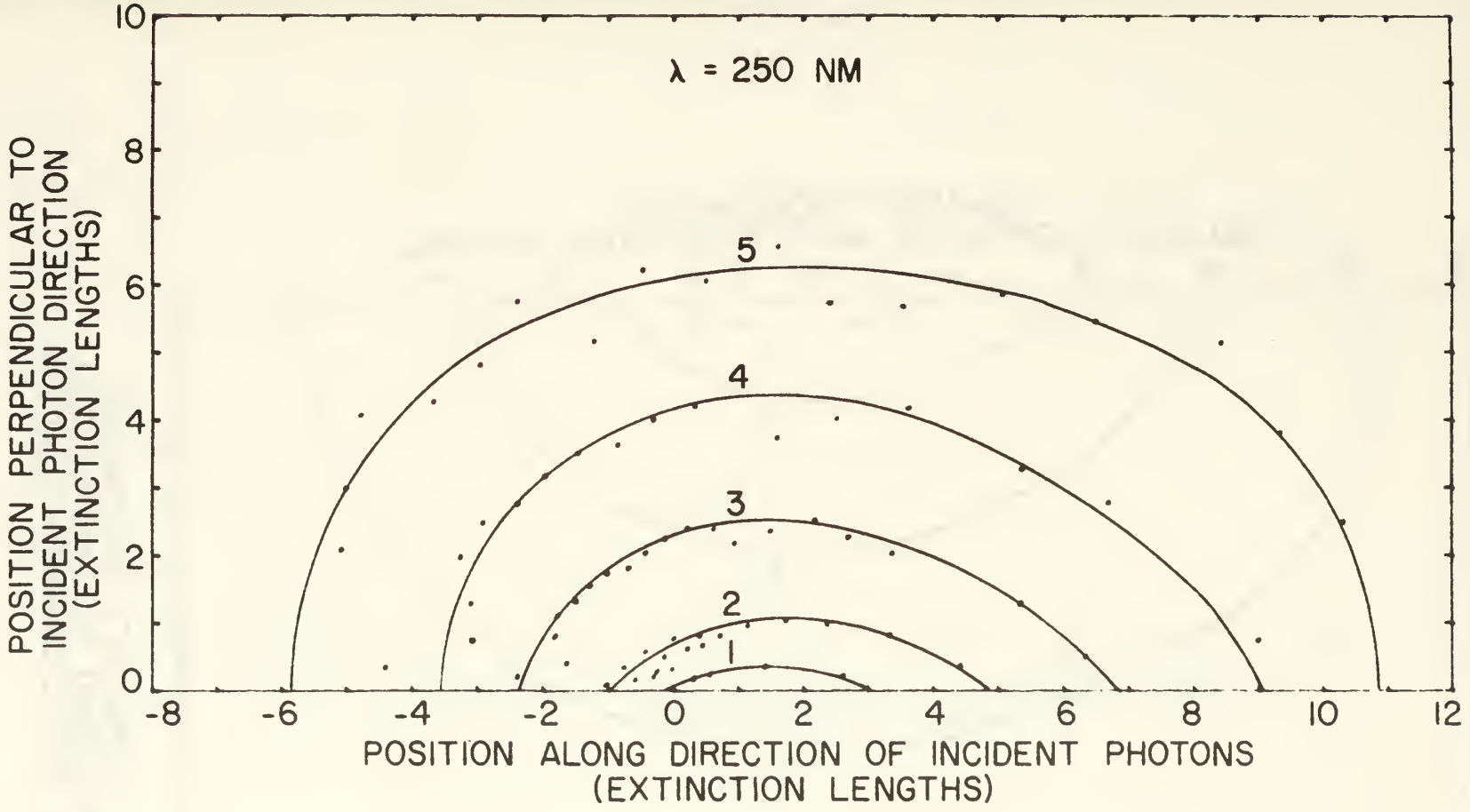

FIGURE 8. Contours of Equal Photon Flux as a Function of the Position of the Observer

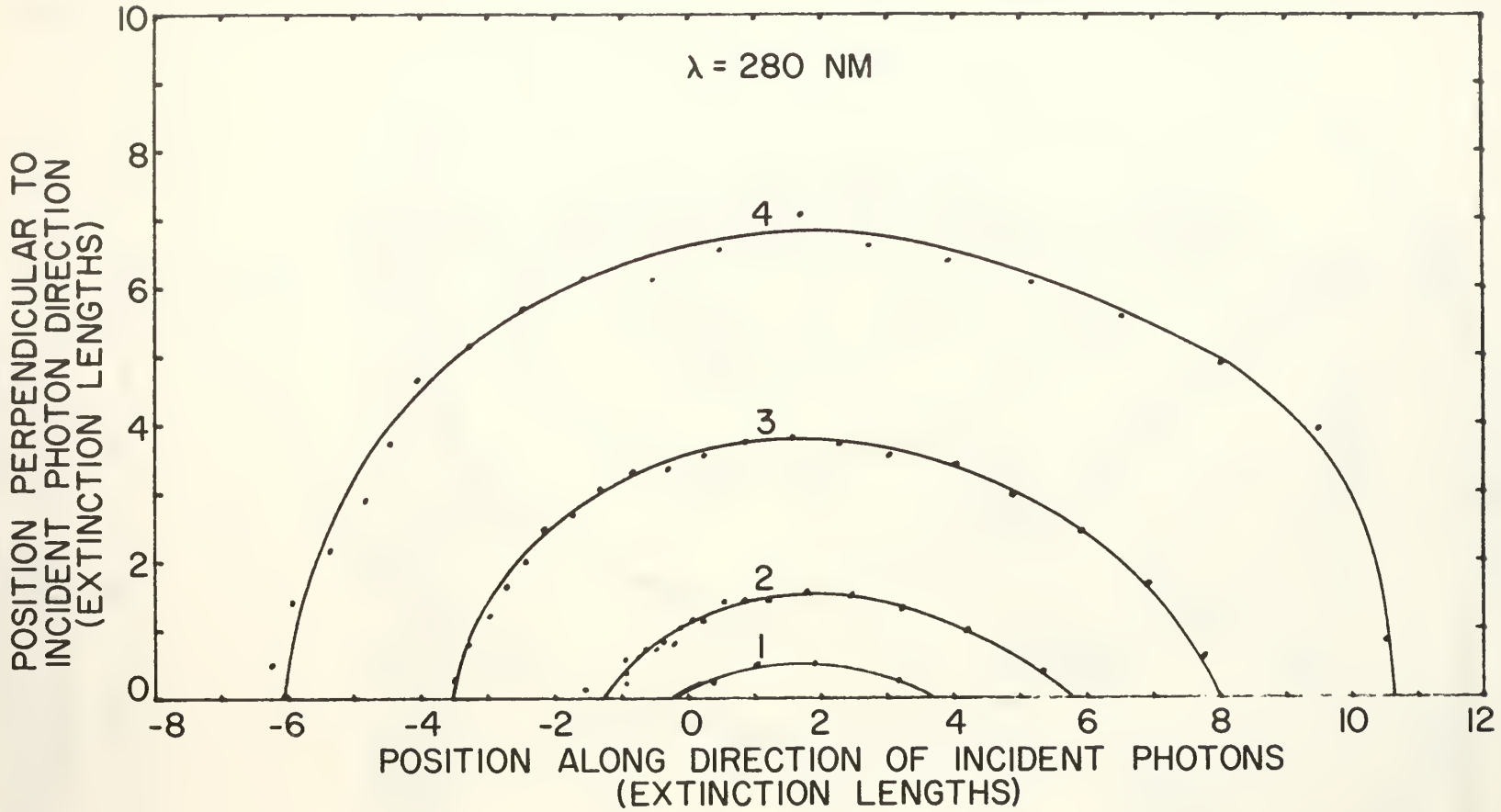

FIGURE 9. Contours of Equal Photon Flux as a Function of the Position of the Observer

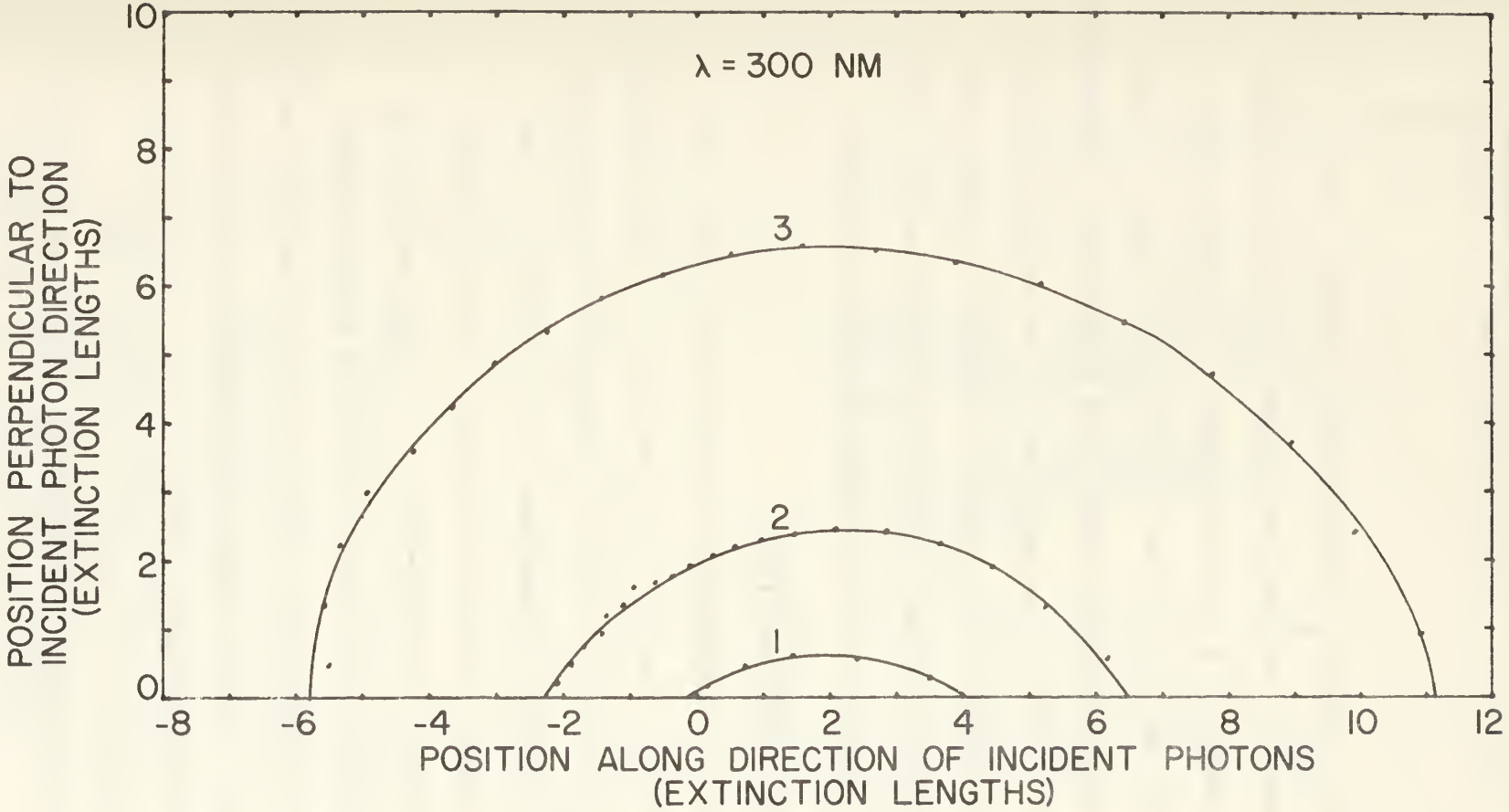

FIGURE 10. Contours of Equal Photon Flux as a Function of the Position of the Observer

$$\text{Relative Flux} = -\log_{10}\left[\frac{N}{N_o\ 2\pi R^2\ (\cos\theta_1 - \cos\theta_2)}\right] \quad (4)$$

where $2\pi R^2\ (\cos\theta_1 - \cos\theta_2)$ is the area of that portion of a hemisphere located R extinction lengths from the source subtended by θ_1 and θ_2, N is the number of photons penetrating that area, and N_o is the number of photons leaving the source. Each successive contour of photon flux represents a decrease of one order of magnitude in the relative photon flux. Thus, for example, the contour labeled 2 gives the location at which the photon flux is 10^{-2} photons per unit area, where unit area is one extinction length squared.

E. FIELD OF VIEW CONSIDERATIONS

Figures 11 and 12 represent the relative number of photons detected at a receiver located zero to nine and 36 to 45 degrees off from the axis of propagation at one and five extinction lengths respectively from a 250 nm laser source. A sixty degree (full cone angle) field of view detector would receive seventy-five to eighty percent of the incident radiation for the θ_1 (zero to nine degrees) and the θ_2 (36 to 45 degrees) cases at one extinction length. At five extinction lengths these percentages tend to remain stable. Thus, given a minimum required photon flux at a detector, it is possible to select the proper field of view for that detector. Conversely, utilizing a fixed

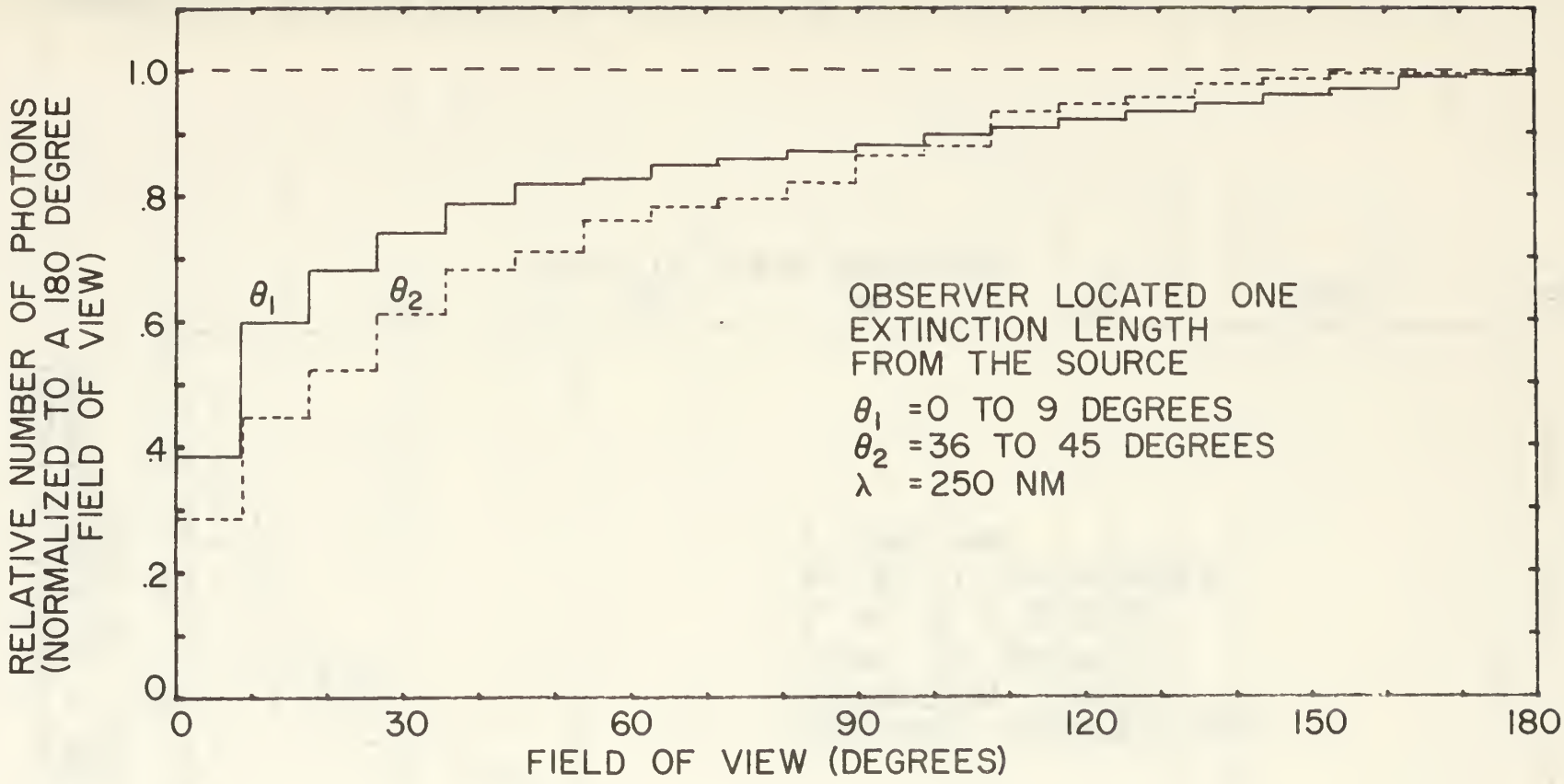

FIGURE 11. Relative Number of Photons vs Field of View for an Observer Located θ Degrees from the Axis of Propagation

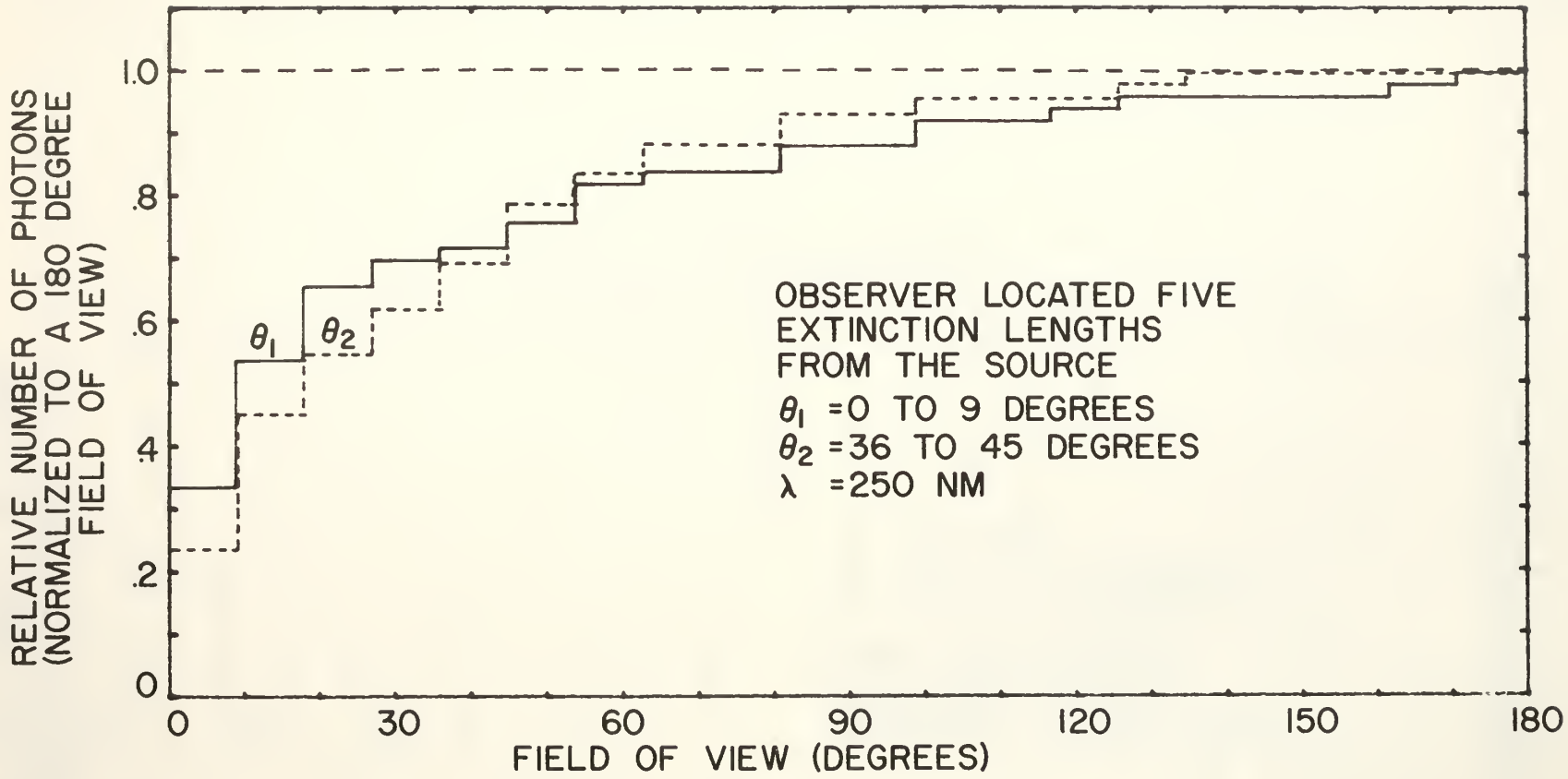

FIGURE 12. Relative Number of Photons vs Field of View for an Observer Located θ Degrees from the Axis of Propagation

field of view detector, it is possible to determine the maximum effective range for a minimum photon flux. (The results for 280 and 300 nm were substantially the same as the 250 nm simulation).

F. PULSE SPREADING CONSIDERATIONS

Figures 13 and 14 represent the pulse spreading as received by a detector located zero to nine and 36 to 45 degrees off from the axis of propagation at one and five extinction lengths from a 250 nm laser source, respectively. The pulse spreading function is a function of the position of the observer and time dispersion caused by multiple scattering. The relative time dispersion, T(R,D) is defined as follows:

$$T(R,D) = \frac{D}{R} - 1 \qquad (5)$$

where D is the total distance traveled by each photon and R is the distance from the source to the shell penetrated by each photon.

The results of pulse spreading at other distances and angles was examined. The pulse spreading function is nearly constant for values of R from one to ten extinction lengths, thus one figure suffices as an approximation to any distance in this angular range. The increase in pulse spreading with increasing angle is illustrated with one additional plot at 36 to 45 degrees off from the

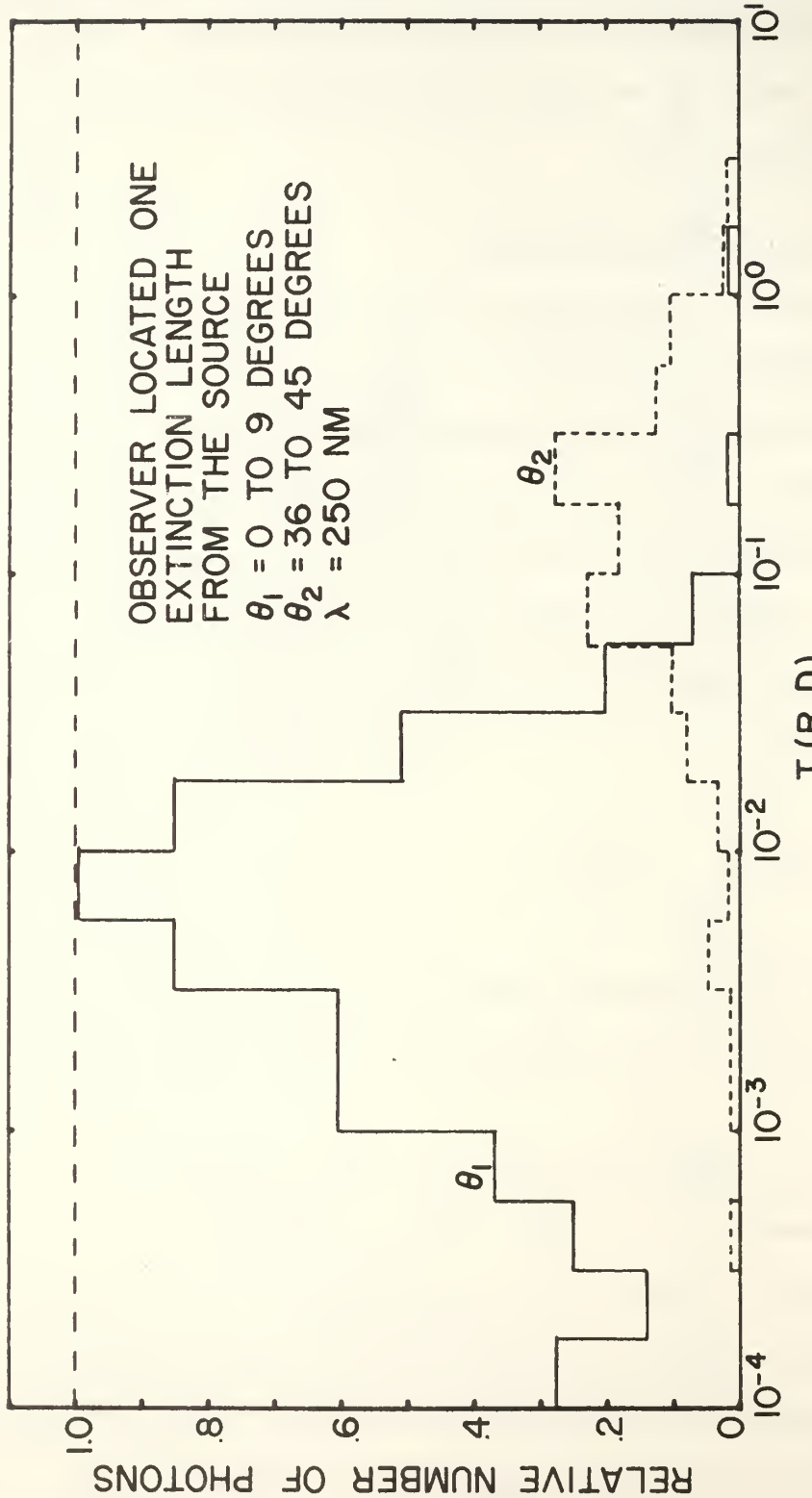

FIGURE 13. Pulse Spreading for an Observer Located θ Degrees from the Axis of Propagation

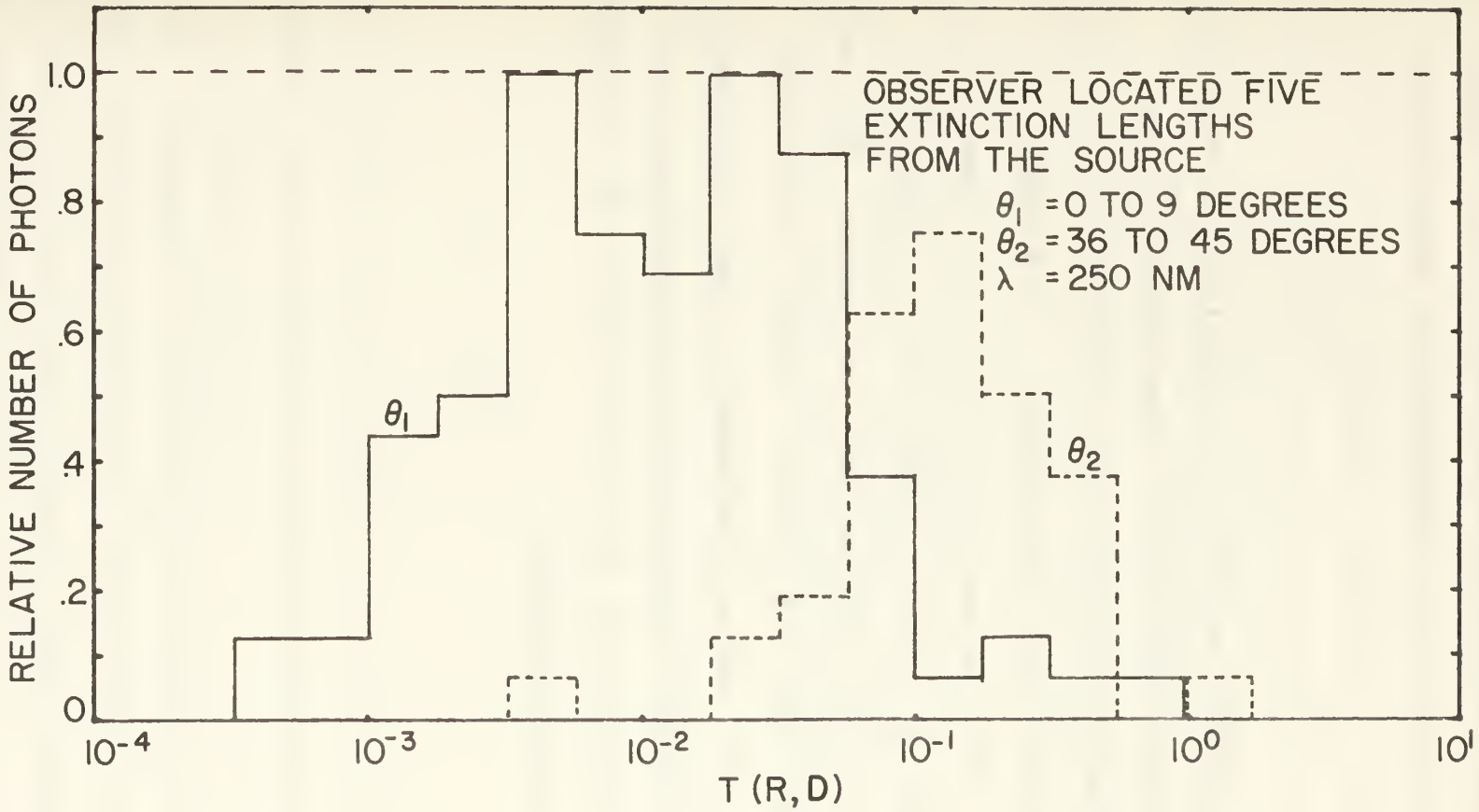

FIGURE 14. Pulse Spreading for an Observer Located Degrees from the Axis of Propagation

axis of propagation. Reproduction of the results at each cone angle introduces a cumbersome presentation of this information, hence only one representative sample is included.

G. SUMMARY

The absorption and scattering coefficients as well as the phase functions from previous experiments were utilized and the behavior of multiple scattered radiation was simulated using the Monte Carlo method. This model was used to study the characteristics of the source beam at a receiver located an arbitrary angle off from the axis of propagation at a given distance from the source. With such information available, the geometric pattern of possible communication links can be determined. A photon density as a function of receiver field of view and position of the observer was determined as well as the effects of multiple scattering on pulse spreading.

VI. SIGNAL-TO-NOISE CONSIDERATIONS

A. INTRODUCTION

The photon flux information presented in the previous section may be utilized to estimate the signal-to-noise ratio (S/N or SNR). This information, however, must be used in conjunction with the characteristics of the detector. Additional considerations therefore include the characteristics of the optical filter, quantum efficiency of the detector, background level of radiation, etc. This section considers the characteristics of available devices and of known atmospheric flux levels in order to estimate the flux level necessary for reliable communication.

B. FUNDAMENTAL RELATIONSHIPS

The detector current generated by a light signal of power P_S and of frequency ν is

$$i_S = \frac{G\eta e P_S}{h\nu} \qquad (6)$$

where G is the gain of the detector, η is the quantum efficiency of the detector, e is the electronic charge, and h is Planck's constant [18].

The dominant noise mechanism for detectors operating in the visible and ultraviolet region of the spectrum is shot noise. The mean square of the noise current due to this noise mechanism is

$$\langle i_n^2 \rangle = 2eG^2[i_d + \frac{\eta e}{h\nu}(P_S + P_B)]\ B \qquad (7)$$

where G is the gain of the detection device (typically a photomultiplier), i_d is the dark current, P_B is the power of the background radiation present due to sources other than that from the desired signal, and B is the bandwidth of the receiver [18, 25, 26, 27].

The desired expression for S/N is thus the ratio of the square of Equations (6) and of (7).

$$S/N = \frac{\langle i_s^2 \rangle}{\langle i_n^2 \rangle} = \frac{(\frac{\eta e P_S}{h\nu})^2}{2e[i_d + \frac{\eta e}{h\nu}(P_S + P_B)]B} \qquad (8)$$

In several specific cases the dark current i_d is negligible, whereupon Equation (8) reduces to Equation (9).

$$S/N \cong \frac{\eta P_S}{2h\nu B}(\frac{P_S}{P_S + P_B}) \qquad (9)$$

For pulsed signals having a time duration of τ, the bandwidth B is effectively $1/2\tau$. The relationship for S/N thus reduces to the number of photons converted to electrons in a pulse, degraded by the ratio of the signal power to total power received over the duration of the pulse.

Each of the expressions for S/N may be modified slightly by consideration of a duty factor, introduced to account for the fact that a signal is not constant over the duration

of a pulse. Consideration of this introduces a multiplicative factor equal to Equation (10).

$$\frac{D_S}{D_N} = \frac{\int_{-\tau/2}^{\tau/2} \left(\frac{i_s(t)}{i_s(o)}\right)^2 dt}{\int_{-\tau/2}^{\tau/2} \left(\frac{i_N(t)}{i_N(o)}\right)^2 dt} \qquad (10)$$

where D_S/D_N is the ratio of the duty factor of the signal to that of the noise over the duration of the pulse, τ.

C. BACKGROUND LEVELS OF RADIATION

The expressions for S/N involve critically the background levels of radiation due to other sources. In communication experiments this is typically due to scattered sunlight. Reliable flux levels at wavelengths longer than 300 nm are reported by Valley [28]. The flux levels at shorter wavelengths are a result of both direct and diffuse transmission. These levels are critically dependent on the thickness of the ozone layer, and have been calculated for various atmospheric conditions [12].

The photon flux calculated by Shettle and Green is presented in Table IV. The dramatic drop in background radiation levels at around 280 nm is evident. The signal that would be transmitted through a narrow band (10 nm) multilayer dielectric filter at 300 nm would include up to 10^{13} photons/cm^2 as background noise photons, whereas at

wavelengths below 280 nm this background radiation level may be considered to be quite small.

TABLE IV

Photon Flux Due to Direct and Diffuse Transmittance through the Earth's Atmosphere [12]

A solar angle of ninety degrees is assumed, along with a 0.32 cm ozone thickness.

λ (nm)	Flux (watts/m^2 nm)	Flux (photons/cm^2 nm sec)
300	1.07×10^{-2}	1.6×10^{12}
295	5.15×10^{-4}	7.8×10^{10}
290	2.14×10^{-6}	3.2×10^{8}
285	9.50×10^{-11}	1.4×10^{4}
280	9.09×10^{-19}	1.4×10^{-4}

D. DETECTOR CHARACTERISTICS

The detector characteristics which relate to a given S/N expression include filter characteristics, quantum efficiency, detector area, and field of view. Using a combination of multilayer dielectric filters and absorption filters, it is possible to construct a device having composite characteristics which will pass ten percent of the incident flux at 265 nm and reject the ambient solar flux in the visible to a degree sufficient to reduce the overall background count level to approximately ten counts per

second [29]. The transmission characteristics of this filter in the 200 to 400 nm region are indicated in Figure 15. It is thus expected that somewhat greater than one percent but less than ten percent of the photons in the 250 to 280 nm region will pass through the filter. Once the photons reach the photocathode of a typical photomultiplier, the conversion efficiency is usually greater than ten percent but less than 25 percent. Thus, for purposes of estimating S/N, the combined effects of filter transmission characteristics plus quantum efficiency of the photocathode may be accounted for by choosing a value of η to be greater than 10^{-3} but less than 10^{-2}. With unforseen developments it is possible to expect a combination having a composite value of 10^{-1}, however such a device is not presently available.

The area of the detector used in the simulation was chosen to be nominally one square inch, typical of currently available photomultiplier tubes.

E. STATISTICAL CONSIDERATIONS

The number of photons converted into an electrical signal follows Poisson statistics:

$$P(r) = e^{-n} n^{r}/r! \qquad (11)$$

where P(r) is the probability that r photons are observed in a sample pulse which has an average number of n photons per pulse.

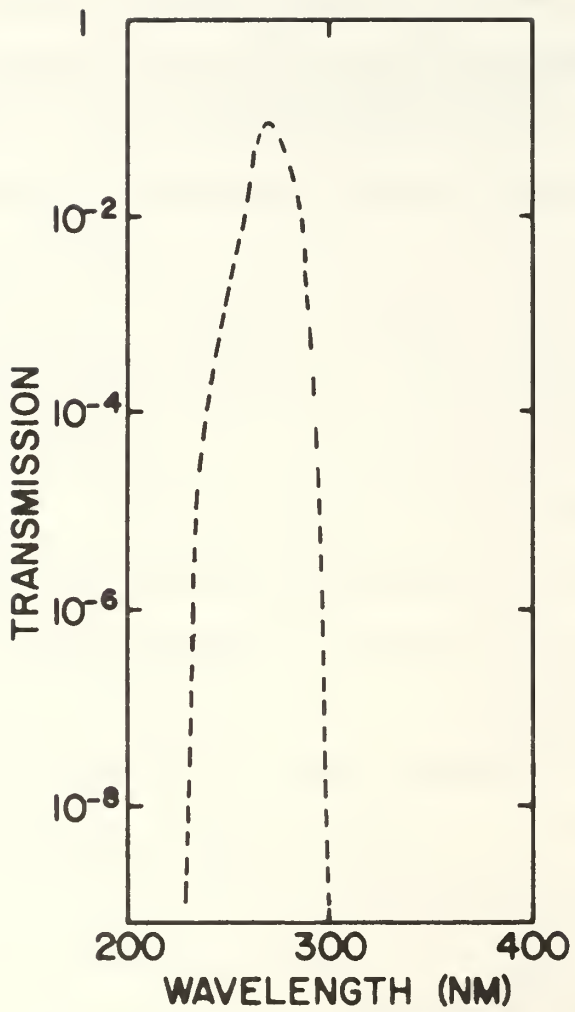

FIGURE 15. Transmission Characteristics for a Typical Composite Filter

The probability of observing an event with a number of photons above the threshold value r_{th} is

$$P(r>r_{th}) = \sum_{r=r_{th}}^{\infty} e^{-n} n^r/r! \qquad (12)$$

Graphs of this function are readily available [30]. Given an average number, n, equal to ten and a threshold level of two (to distinguish from the random single photon events which occur at the detector with a frequency of occurrence equal to ten or twenty photons per second), a probability of detection of such pulses is .9995. If the average number n is five with the threshold number equal to two, the probability of detection is 0.96. Thus an average value of ten photons per pulse in the absence of background radiation is chosen as a desired level to avoid excessive error. This means that a desirable value of S/N is ten.

F. SUMMARY OF SIGNAL-TO-NOISE CONSIDERATIONS

For the case of minimal background radiation, Eq. (13) gives the relationship for the number of signal photons required.

$$\frac{P_s \tau}{h\nu} = \frac{1}{\eta} \left(\frac{S}{N}\right) \qquad (13)$$

If background radiation dominates the power incident on the detector (as would be the case at wavelengths longer than 300 nm), then the number of signal photons required is given by Eq. (14).

$$\frac{P_s \tau}{h\nu} = \left[\frac{1}{\eta} \left(\frac{S}{N}\right) \left(\frac{P_B \tau}{h\nu}\right)\right]^{1/2} \qquad (14)$$

Table V gives a summary of the necessary photon flux external to the detector in order to obtain a S/N equal to ten. These flux levels may be used in conjunction with the figures of Section V to generate the geometric patterns over which signal transmission may be expected.

TABLE V

Photon Flux Required in order to Give a S/N Equal to Ten

Assumed parameters:

Filter characteristics for Figure 15

Quantum efficiency of ten percent

$\tau = 10^{-6}$ seconds (significant pulse spreading)

Area of detector is one square inch

Note: Filter characteristics at 300 nm have been assumed for a narrow band (ten nm) multilayer dielectric filter.

λ (nm)	η	$\frac{P_B}{h\nu}$ ($\frac{\text{photons}}{\text{sec}}$)	$\frac{P_S \tau}{h\nu}$ (photons)	Required flux (photons/cm^2)
250	10^{-3}	---	10^4	1.6×10^3
265	10^{-2}	---	10^3	1.6×10^2
280	10^{-3}	---	10^4	1.6×10^3
300	10^{-2}	10^{13}	10^5	1.6×10^4

VII. DISCUSSION

Although not currently available, three millijoule laser sources of different wavelengths (250, 280, and 300 nanometers) capable of being pulsed at an appropriate rate (200 pulses per second for 2400 bit per second data rate using 12 bits per pulse [4]) were assumed. (The navy has used vocoders with 2400 bits per second capability since the 1940's). Table V lists the assumptions leading to the required photon flux in order to achieve a S/N equal to ten. By utilizing the contours of equal photon flux in Figures 8, 9, and 10, the ranges for which pulse position modulation communication is possible were predicted. Table VI lists the results of these calculations.

Predicted performance for lasers operating at 250 and 280 nm are comparable for sixty degree or greater off axis detection. For detection angles less than sixty degrees the 280 nm laser is superior. The performance of the 300 nm laser does not appear to be degraded by the background radiation as much as might be expected. The performance for this laser from zero to twenty degrees off from the axis of propagation is similar to the 280 nm laser. Beyond thirty degrees the ranges decrease markedly, indicating very little backscattering. It cannot be overstressed that the results above are for a simulation of assumed laser sources with assumed atmospheric propagation

TABLE VI

Predicted Ranges or Range Bands for Communication
in the Middle UV for Pulsed Millijoule Lasers

(θ is the angular location of the receiver
with respect to the axis of propagation.)

λ = 250nm ($1.6 \times 10^3 \frac{\text{Photons}}{\text{cm}^2}$ req'd) λ = 280nm ($1.6 \times 10^3 \frac{\text{Photons}}{\text{cm}^2}$ req'd) λ = 300nm ($1.6 \times 10^4 \frac{\text{Photons}}{\text{cm}^2}$ req'd)

	$5000 \frac{\text{Photons}}{\text{cm}^2}$	$500 \frac{\text{Photons}}{\text{cm}^2}$		$1526 \frac{\text{Photons}}{\text{cm}^2}$		$7875 \frac{\text{Photons}}{\text{cm}^2}$
θ (Degrees)	Range (km)	Range (km)	θ (Degrees)	Range (km)	θ (Degrees)	Range (km)
0	2.4	3.4	0	5.5	0	5.2
30	1.1	2.1	30	2.9	10	3.6
60	.52	1.7	60	1.6	20	2.2
90	.34	1.2	90	1.0	30	.93
120	.30	1.0	120	.92	40	.24
150	.33	1.1	150	.94	90	.10
180	.49	1.2	180	1.2	180	.10

(Ranges for λ = 250 nm should be approximately half-way between the ranges listed.)

(Ranges for λ = 280 nm should be on the order of the range listed.)

(Ranges for λ = 300 nm should be somewhat less than the range listed.)

characteristics and assumed detector characteristics. Clearly, in order to make accurate predictions for real systems it is necessary to obtain accurate atmospheric characteristics as well as other system parameters.

VIII. CONCLUSION

Lasers operating in the middle uv do not appear to be suitable for long range communications; however, they present definite possibilities for short range (several km) applications. Since radiation in this region is absorbed exponentially, rather than as the inverse of distance squared, the potential for covertness is highly accentuated. Current lasers, however, having a power output on the order of 0.1 joules or greater with low pulse rates (15 to 20 pulses per second), are not suitable for voice communication due to the low repetition rates. The pulsed mode of operation appears to hold definite advantages over amplitude modulation of a continuous wave laser from simple signal-to-noise considerations. Good possibilities for successful communication are offered by pulse position modulation. Millijoule laser sources operating with 200 or more pulses per second are desirable but not currently available.

The contours of equal photon flux presented in this paper are useful for determining the communication patterns of possible laser sources. The pulse spreading and field of view figures are likewise useful for predicting the characteristics of proposed receivers. A multiple scattering model is definitely required for proper prediction of photon flux over expected distances of communication.

APPENDIX A

MONTE CARLO SIMULATION OF THE MULTIPLE SCATTERING PROBLEM

A. FUNDAMENTAL RELATIONSHIPS

1. Weighting Factors for Monte Carlo Calculations

The following calculations served as a model for the derivation of the various weighting factors:

PROBLEM: Given a random number generator that provides numbers within a specified interval along the x-axis (with equal probability for each interval dx) obtain random numbers with a probability p(y). I.e., Find y(x) such that $p(y)dy = dx$.

SOLUTION: Let Random Variable Y be distributed in $y_o \leq y \leq y_2$ and its density function be given by

$$p(y) = \begin{cases} p(y) & y_o \leq y \leq y_2 \\ 0 & \text{otherwise} \end{cases} .$$

The distribution function, P(y), is given by

$$P(y) = P(Y \leq y) = \int_{-\infty}^{y} p(y)\, dy$$

Therefore, $P(y) = \int_{y_o}^{y} p(y)\, dy$, since $p(y) = 0$ for $y < y_o$.

Now,

$$p(y)\, dy = dx$$

Integrating,

$$y(x) = y_o + P^{-1}(\Delta x)$$

where

$$y_o = y(x_o)$$

and $$\Delta x = x - x_o$$

This concept applied to specific cases yields the following fundamental relationships for weighting factors:

a. Exponential

$$p(y) = \frac{1}{\tau} e^{-y/\tau} \qquad (0 \leq y \leq \infty)$$

$$y(x) = -\tau \ln (1 - \Delta x)$$

b. Henyey-Greenstein

$$p(\theta) = \frac{(1-g^2)}{2(1+g^2-2g \cos \theta)^{3/2}} \qquad (0 \leq \theta \leq \pi)$$

where

$$g = \langle \cos \theta \rangle$$

$$\theta(x) = \cos^{-1} \left\{ \frac{1+g^2}{2g} - \frac{1}{2g} \left[\frac{1-g^2}{2g(\frac{1+g}{2g} - \Delta x)} \right]^2 \right\}$$

c. Modified Henyey-Greenstein [Zachor 1977]

$$p(\theta) = \frac{1-g^2}{2}\left[\frac{1}{(1+g^2-2g\cos\theta)^{3/2}} + \frac{f}{2}\frac{(3\cos^2\theta-1)}{(1+g^2)^{3/2}}\right]$$

$$0 \leq \theta \leq \pi$$

where

$g = \langle\cos\theta\rangle$ and f is a weighting factor.

$$\theta(x) = \cos^{-1}\left\{\frac{1+g^2}{2g} - \left[\frac{(g^2-1)^2}{8g^3}\right]\frac{1}{[\Delta x - F(\theta) - \frac{1+g}{2g}]^2}\right\}$$

where

$$F(\theta) = \left[\frac{f}{4}\frac{(1-g^2)}{(1+g^2)^{3/2}}\right](\cos\theta - \cos^3\theta)$$

d. Uniform

$$p(\phi) = \frac{1}{2\pi} \qquad 0 \leq \phi \leq 2\pi$$

$$\phi(x) = 2\pi\,\Delta x$$

e. Rayleigh

$$p(\theta) = \frac{3}{8}(1 + \cos^2\theta) \qquad 0 \leq \theta \leq \pi$$

$$\theta(x) = \cos^{-1}(A + B)$$

where

$$A = \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + 1}}$$

$$B = \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + 1}}$$

and

$$b = 8\,\Delta x - 4$$

2. Position and Coordinates in Photon-Fixed Coordinate System

a. Axis Rotation

Axis rotation is accomplished by the following transformation:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\Delta\theta\cos\Delta\phi & -\cos\Delta\theta\sin\Delta\phi & -\sin\Delta\theta \\ \sin\Delta\phi & \cos\Delta\phi & 0 \\ \sin\Delta\theta\cos\Delta\phi & -\sin\Delta\theta\sin\Delta\phi & \cos\Delta\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

b. Axis Translation

Axis translation is accomplished by the following transformation:

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \Delta r \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

3. Angles $(\theta,\phi,\theta',\phi')$ for a Photon Passing Through a Shell[1]

A laser is located at the origin of the $\hat{i}_o$, $\hat{j}_o$, $\hat{k}_o$ coordinate system. The initial direction of propagation of the collimated beam is along the $-\hat{k}_o$ axis. θ is the angle between the position vector $\overline{R}$ (pointing from the location where the photon penetrates a shell toward the laser source) and the vector $-\hat{k}_o$. ϕ is the angle between the projection of $-\overline{R}$ onto the $(\hat{i}_o,\hat{j}_o)$ plane (denoted by $-\overline{R}_p$) and the vector $\hat{i}_o$.

A photon is located at the origin of the $\hat{V}_x$, $\hat{V}_y$, $\hat{V}_z$ and the $\hat{U}_x$, $\hat{U}_y$, $\hat{U}_z$ coordinate systems. As the photon passes through the shell, θ' specifies the angular direction of photon travel (along $\hat{U}_z$) relative to $\overline{R}$ (the line-of-sight position vector). ϕ' is the angle between the projection of $\hat{U}_z$ (denoted by the vector $\overline{W}$) onto the $(\hat{V}_x,\hat{V}_y)$ plane and $\hat{V}_x$.

The vectors $\overline{R}_i$, $\overline{R}_j$, and $\overline{R}_k$ are position vectors in the $\hat{U}_x$, $\hat{U}_y$, $\hat{U}_z$ coordinate system denoting the location of the points (1,0,0), (0,1,0), and (0,0,1) in the $\hat{i}_o$, $\hat{j}_o$,

---

[1]See Figure 16.

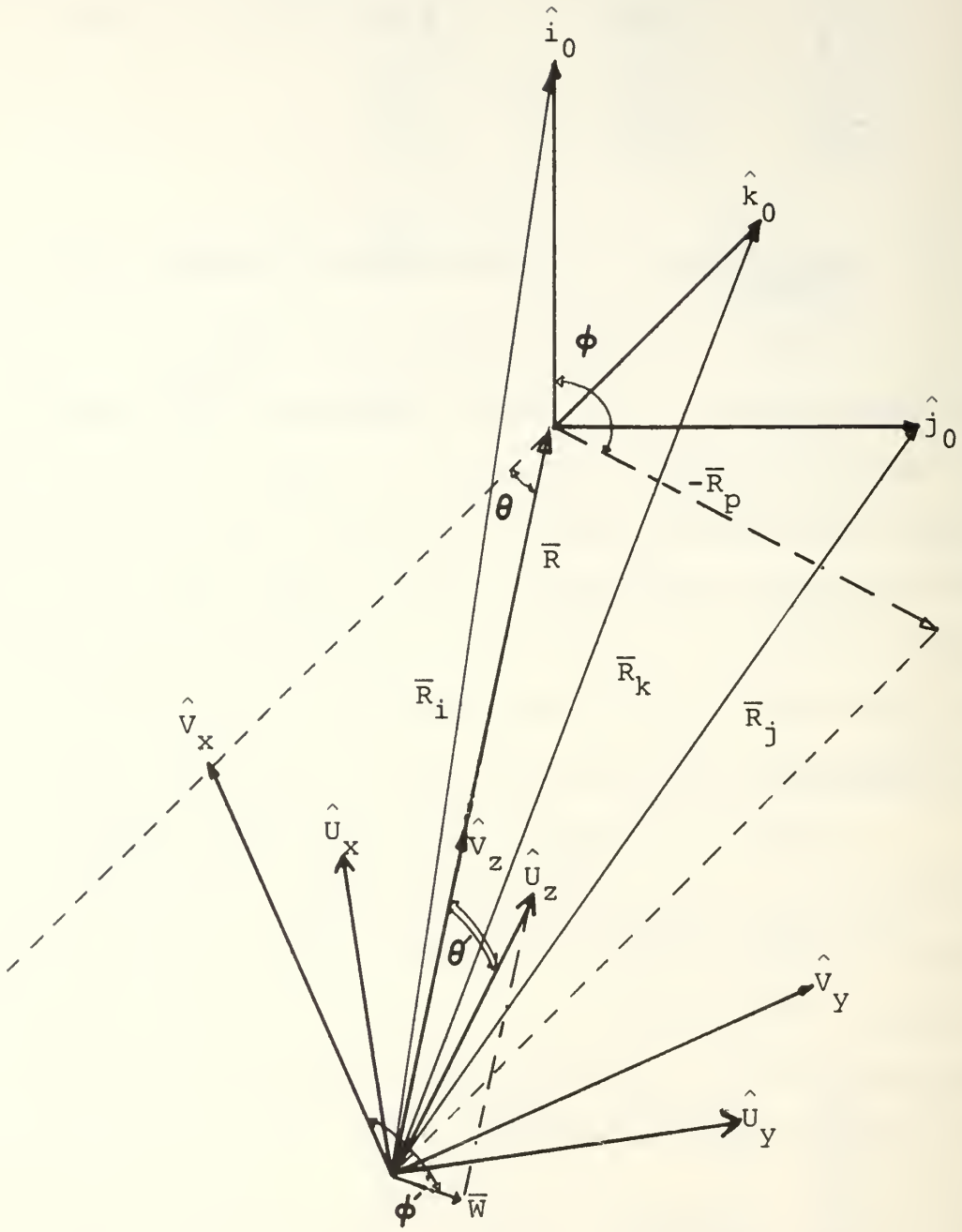

FIGURE 16. Diagram of the Photon Fixed Coordinate System

$\hat{k}_o$ coordinate system. (These points are used for orientation purposes.)

a. $$\theta = \cos^{-1}\left(\frac{\bar{R} \cdot \hat{k}_o}{|\bar{R}| \quad |\hat{k}_o|}\right)$$

$$\theta = \cos^{-1}\left[\frac{\bar{R} \cdot (\bar{R}_k - \bar{R})}{r}\right] \qquad (0 \leq \theta \leq \pi)$$

b. $$\phi = \cos^{-1}\left(\frac{\bar{R} \cdot \hat{i}_o}{|\bar{R}_p|}\right)$$

$$\phi = \cos^{-1}\left[\frac{\bar{R} \cdot (\bar{R}_i - \bar{R})}{r_p}\right]$$

where

$$r_p = \sqrt{r^2 - [(\bar{R}_K - \bar{R}) \cdot \bar{R}]^2} \qquad (0 \leq \phi \leq 2\pi)$$

c. $$\theta' = \cos^{-1}\left(\frac{\hat{U}_z \cdot \bar{R}}{|\bar{R}|}\right)$$

$$\theta' = \cos^{-1}\left(\frac{\hat{U}_z \cdot \bar{R}}{r}\right) \qquad (0 \leq \theta' \leq \pi)$$

d. $$\phi' = \cos^{-1}\left(\frac{\hat{V}_x \cdot \bar{W}}{w}\right) \qquad (0 \leq \phi' \leq 2\pi)$$

where

$$w = 1 - (\hat{V}_z \cdot \hat{U}_z)^2$$

4. Mapping Spherical Coordinates onto Polar Coordinates and Coordinate Transformation

In order to map a point on a sphere onto a circle(s) such that there is a one-to-one mapping the following transformations were utilized:

Case I. $0 \leq \theta \leq \pi/2$

$$r = R\sqrt{1 - \cos\theta}$$

$$\theta = \cos^{-1}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Case II. $\pi/2 \leq \theta \leq \pi$

$$r = R\sqrt{1 + \cos\theta}$$

$$\theta = \cos^{-1}\left[\left(\frac{r}{R}\right)^2 - 1\right]$$

In order to plot (θ,ϕ) the following coordinate transformation was used:

I.e. - Given (θ,ϕ), find (x,y).

$$r = \begin{cases} R\sqrt{1 - \cos\theta} & 0 \leq \theta \leq \pi/2 \\ R\sqrt{1 + \cos\theta} & \pi/2 \leq \theta \leq \pi \end{cases}$$

$$x = r\cos\phi$$

$$y = r\sin\phi$$

5. Phase Functions

A factor of $(\frac{1}{2\pi})$ arises from the normalization process for the phase function.

$$\text{(i.e. } \int_0^{2\pi} \int_0^{\pi} p(\theta) \sin\theta \, d\theta \, d\phi = 1\text{)}$$

a. Particulate

$$P(\theta) = \frac{1-g^2}{4\pi}\left[\frac{1}{(1+g^2-2g\cos\theta)^{3/2}} + \frac{f}{2}\frac{3\cos^2\theta-1)}{(1+g^2)^{3/2}}\right]$$

b. Rayleigh

$$P(\theta) = \frac{3}{16\pi}(1+\cos^2\theta)$$

APPENDIX B

DEVELOPMENT OF THE FUNDAMENTAL RELATIONSHIP FOR RAYLEIGH SCATTERING

$$p(\theta) = \frac{3}{8}(1 + \cos^2\theta) \qquad 0 \leq \theta \leq \pi$$

$$x = \int_0^{\theta} p(\theta)\, d\cos\theta$$

$$= \int_0^{\theta} \frac{3}{8}(1 + \cos^2\theta)\, d\cos\theta$$

$$= -\frac{3}{8}\left(\cos\theta + \frac{\cos^3\theta}{3}\right)\Big|_0^{\theta}$$

$$x = -\frac{3}{8}\left(\cos\theta + \frac{\cos^3\theta}{3}\right) + \frac{1}{2}$$

Now solve for $\cos\theta$.

$$\cos^3\theta + 3\cos\theta + (8x - 4) = 0$$

Let

$$y = \cos\theta$$

$$b = 8x - 4$$

$$a = 3$$

$$y^3 + ay + 6 = 0$$

(See Reference 31 for the solution to a cubic equation.)

$$y = A + B$$

or

$$y = -\frac{A + B}{2} + \frac{A - B}{2}\sqrt{-3}$$

or

$$y = -\frac{A + B}{2} - \frac{A - B}{2}\sqrt{-3}$$

where

$$A = \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}$$

and

$$B = \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}$$

Now

$$\frac{b^2}{4} + \frac{a^3}{27} = \frac{b^2}{4} + \frac{3^3}{27} = \frac{b^2}{4} + 1$$

$$\frac{b^2}{4} \geq 0; \quad \frac{b^2}{4} + 1 > 0$$

Therefore, only one real root and two conjugate imaginary roots exist.

Since $y = \cos\theta$, only one real root is considered.

$$\theta = \cos^{-1}(A + B)$$

where

$$A = \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + 1}}$$

$$B = \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + 1}}$$

and

$$b = 8x - 4$$

APPENDIX C

SAMPLE CALCULATION FOR RANGE PREDICTION

PROBLEM: Given a single channel 280 nm laser utilizing pulse position with power output per pulse equal to one millijoule, determine the communication pattern for a S/N equal to ten. Assume the detector characteristics presented in Section VI.

SOLUTION: From Table V, the minimum photon flux for a S/N of ten is 1.6×10^3 photons per cm^2.

Now,

$$1 \text{ watt} = 5.0345 \times 10^{18} \lambda \frac{\text{photons}}{\text{sec}}$$

where

$\lambda = .280\ \mu$ (λ is measured in μ)

$$1 \text{ joule} = 1 \text{ watt sec}$$

$$= 1.40966 \times 10^{18} \text{ photons}$$

$$1 \text{ millijoule} = 1.40966 \times 10^{15} \text{ photons}$$

From Figure 9, at contour 2 the relative flux is 10^{-2} of the initial photons per unit area where the unit area is one extinction length squared. From Table III one extinction length is

$(1.0405\ km^{-1})^{-1}$ or .9611 kilometers. The relative flux is $1.0826 \times 10^{-12}\ cm^{-2}$. The irradiance is given by the product of 1.40966×10^{15} and 1.0826×10^{-12} or 1526 photons per square cm. By measuring the distance from the source to the detector located on contour 2 (Figure 9) and recalling that one extinction length for this case is .9611 km, the ranges for an irradiance of 1526 photons/cm^2 may be determined. Table VI lists the results of this calculation.

APPENDIX D

CHECKS ON POSSIBLE ERRORS

Each computational procedure involved in writing this program was verified and an effort was made to ensure the entire program functioned properly. The procedures utilized to document the correct behavior of this routine are described in this appendix.

The various weighting functions in the program were verified by comparing the actual distribution of photons generated by the computer to the analytical solution for the distribution of random numbers obtained using a programmable calculator. (The statements required for this generation remain in the program, but the output of the results is suppressed.)

Verification of axis transformation computations was simple and straight-forward. The axes were rotated about the Z axis, the Y axis, and finally about both axes to ensure proper behavior. (No documentation steps of this nature have been left in the program.) The output of θ, ϕ,θ', and ϕ' for each photon at each shell was ordered by a computer sort routine in order to facilitate the preparation of a scatter diagram indicating the pattern formed by the simulated photons. Inspection of such a pattern did in fact reveal the presence of a fault which was corrected.

The results of Duntley [32] in describing multiply scattered radiation in the propagation of laser light under water are most useful. Flux as a function of cone angle is presented for distances of up to 19 attenuation lengths. The scattering phase function for light underwater is a sharply peaked function which may be approximated with a Henyey-Greenstein function with a g value equal to 0.96 [33]. Calculations of photon flux as a function of cone angle were carried out with the computer program developed. Total attenuation of the beam down to a level of 10^{-3} could be calculated with some measure of accuracy (only 10,000 simulated photons were utilized for these calculations.) The results of these calculations agreed with the behavior observed in these underwater scattering measurements, simulating correctly the flux levels out to 16 attenuation lengths (the limit calculated).

Further verification of the accuracy of the program was obtained by comparison with calculations by Zachor and Green [24]. In this article, photon flux as a function of viewing angle was calculated for several distances from the source. The same conditions were assumed with the model used here, resulting in satisfactory agreement with the information published by Zachor for distances of 1.43, 2.73, and 4.95 extinction lengths for 300 nm radiation.

APPENDIX E

PROGRAM DESCRIPTION AND DOCUMENTATION

A. COMPONENTS OF THE PROGRAM

1. Weighting Function Subprograms

RANEXP is a function subprogram that generates random numbers weighted exponentially. It is used for distance calculations.

Function subprogram RANTH generates random values for theta and theta prime. Since the interaction may be either Rayleigh scattering or particulate scattering, either calculation may be performed. A uniform random number (generated by RANDU) within the interval [0,1] is obtained and compared to R_P (the ratio of particulate to total scattering). If the random number is less than R_P the particulate scattering calculation is made. (Otherwise the Rayleigh calculation is performed.) Due to the complexity of the particulate phase function it is difficult to obtain an exact solution; therefore, an iterative method is employed. An exact solution is available when f is set to zero and a more efficient calculation is utilized.

RANPH generates a random value of phi or phi prime uniformly weighted on the interval $[0,2\pi]$. The steps in this function subprogram are trivial.

2. LITE Subroutine

The main subroutine of the Monte Carlo simulation, called LITE, simulates a photon which randomly interacts with atmospheric constituents. (In order to make the most efficient use of computer time, non-interaction was not considered.) Initially the photon travels along the $-K_0$ axis (see Figure 16, Appendix A), a distance ΔR (determined by function subprogram RANEXP). A uniform random variable in the interval [0,1] is compared to R_S (the ratio of the total scattering coefficient, K_S, to the total extinction coefficient, K). If the random number is smaller than R_S, the photon is scattered. (Otherwise it is absorbed and the photon is lost.) If it is scattered, the distance of the photon from the laser is updated and a new value of R is calculated. Since the photon has been scattered, new angles and distances must be computed. This is accomplished by generating values for $\Delta\theta$, $\Delta\phi$, and ΔR (using RANTH, RANPH, and RANEXP, respectively). The photon coordinate system is transformed (translated and rotated) to a new coordinate system. The distance from the source is calculated and compared with the radius of a shell. (A series of concentric shells with radii measured in extinction lengths, may be formed in order to determine the photon flux or angular distribution of photons as a function of distance from the source.) Various calculations are then made to determine whether a photon has penetrated through a shell, passed in and out of a shell, or penetrated an outer shell. These calculations are necessary to ensure proper accounting of

the photon flux. When a photon passes through a shell, the position of the source with respect to the penetration point is obtained. Theta, phi, and the radius of the shell (measured in extinction lengths) are stored in a three dimensional array for future use. Theta prime and phi prime are also determined as a function of distance (using RANTH and RANPH). These too, are stored in a three dimensional array and specify the direction of incident photon flux as observed by a receiver located a given distance from the source at a given angle off from the axis of propagation. This process is repeated until all of the photons have been used to generate information. (It should be noted that many extra steps incorporated to debug the program, remain in to facilitate future implementation by another user.)

3. DRLITE Routine

The routine DRLITE is used to drive the subroutine LITE. Information is read into the computer and the results are written out using this routine. The spatial distribution of photons as a function of position and detector field of view is calculated as well as the total number of photons in each shell. The output of other available information is optional and is controlled by IPRT and METHOD statements. This information includes pulse spreading (or photon time of arrival) as a function of position and field of view, the relative photon flux at each annular ring in a shell,

and a least square curve fitting of a modified Henyey-Greenstein function to the phase function information generated by the computer.

4. Miscellaneous Subroutines and Function Subprograms

LSTHG, LEAST, GAUSS3, and RANDU are miscellaneous subroutines used in the program. LSTHG is used to least squares fit a modified Henyey-Greenstein function to the information generated. LEAST and GAUSS3 are used in conjunction with LSTHG. EQN is a function subprogram supporting LSTHG. RANDU is a uniform random number generator that provides numbers in the interval [0,1]. It is located in the computer library. The variable IX is the seed and may be any odd integer with nine digits or less. It may not be zero. In order to use RANDU the following FORTRAN statements are used:

```
IX = 948752759
CALL RANDU (IX,IY,Y)
IX = IY
```

The seed IX is used in the calculation and a new seed IY is generated for future use. A uniform random number, Y, is in the interval [0,1] and is also available for future calculations. The last statement (IX = IY) resets the seed IX to a new value.

```
C     THIS ROUTINE IS INTENDED TO DRIVE THE MONTE CARLO ROUTINE          DRL00010
C     CALLED LITE, WHICH CALCULATES THE DISTRIBUTION OF PHOTONS          DRL00020
C     FROM A UNIDIRECTIONAL LIGHT SOURCE.  IT IS USED TO READ            DRL00030
C     INFORMATION INTO THE PROGRAM AND TO CONTROL THE OUTPUT.            DRL00040
C     THE DISTRIBUTION OF PHOTONS AND THE TOTAL NUMBER OF PHO-           DRL00050
C     TONS IN EACH SHELL ARE STANDARD OUTPUT INFORMATION.                DRL00060
C     OPTIONAL OUTPUT IS CONTROLED BY METHOD AND IPRT STATE-             DRL00070
C     MENTS.  (METHOD CONTROLS THE SHAPE OF THE THETA BINS,              DRL00080
C     WHILE IPRT CONTROLS THE ACTUAL OUTPUT).                            DRL00090
C                                                                        DRL00100
C          IPRT = 0,1,2,3,4,5,6,7,8, OR 9 YIELDS THE STANDARD            DRL00110
C                 OUTPUT.                                                DRL00120
C          IPRT = 1,2,3,4, OR 5 YIELDS THE NEGATIVE LOG OF               DRL00130
C                 RELATIVE FLUX AT EACH ANNULAR RING.                    DRL00140
C          IPRT = 3,4,5,6,7,8,OR 9 YIELDS THE TIME OF ARRIVAL            DRL00150
C                 OR PULSE SPREADING INFORMATION.                        DRL00160
C          IPRT = 6,7,8,OR 9 YIELDS A LEAST SQUARE FIT TO A              DRL00170
C                 MODIFIED HENYEY-GREENSTEIN FUNCTION TO THE             DRL00180
C                 PHASE FUNCTION INFORMATION.                            DRL00190
      REAL*8 GIN                                                         DRL00200
      INTEGER*4 BINS(20,50,1), BINDST(20,50,20)                          DRL00210
      REAL*4 RFLUX(20,50,1)                                              DRL00220
      EQUIVALENCE (BINS,RFLUX)                                           DRL00230
      DATA PI/3.14159265367                                              DRL00240
    1 WRITE(7,500)                                                       DRL00250
  500 FORMAT(' NPHOT, NTHETA, NFLDVW, NSHLS, METHOD, IX, DISTSH, RATIO,  DRL00260
     * G, RPT, RBACK, GIN',/,      '     I5     I2     I2     I2     I2  DRL00270
     *I10  5F5.4,F5.2')                                                  DRL00280
      READ(5,505) NPHOT, NTHETA, NFLDVW, NSHLS, IPRT, METHOD, IX,        DRL00290
     * DISTSH, RATIO,  G, RPT, RBACK, GIN                                DRL00300
  505 FORMAT(I5,I2,2I2,2I1,I10,5F5.2,F5.2)                               DRL00310
      IF(METHOD.EQ.0) METHOD = 1                                         DRL00320
      IF(NPHOT.EQ.0) STOP                                                DRL00330
      WRITE(6,110)                                                       DRL00340
  110 FORMAT(1H1)                                                        DRL00350
      CALL LITE(NPHOT,NTHETA,NFLDVW,NSHLS,DISTSH,RATIO,G,RPT,RBACK,      DRL00360
     *  IX,BINS,BINDST,METHOD,IPRT)                                      DRL00370
      WRITE(6,519)                                                       DRL00380
  519 FORMAT(/' DISTRIBUTION OF PHOTONS IS GIVEN BY')                    DRL00390
      DO 50 I = 1,NSHLS                                                  DRL00400
      ISUM = 0                                                           DRL00410
      DO 44 J = 1,NTHETA                                                 DRL00420
      DO 44 K = 1,NFLDVW                                                 DRL00430
      ISUM = ISUM + BINS(I,J,K)                                          DRL00440
   44 CONTINUE                                                           DRL00450
      WRITE(6,520)                                                       DRL00460
  520 FORMAT(1X)                                                         DRL00470
      WRITE(6,440) I,ISUM                                                DRL00480
```

```
  44C FCRMAT(1X,'TOTAL NUMBER IN SHELL ',I2,' IS ',I6)                    DRL00490
      IF(NFLDVW.EC.1) GC TO 47                                            DRL00500
      DC 46 J=1,NTHETA                                                    DRL00510
      ISUM = 0                                                            DRL00520
      DC 45 K=1,NFLDVW                                                    DRL00530
   45 ISUM = ISUM + BINS(I,J,K)                                           DRL00540
      WRITE(6,441) J,ISUM                                                 DRL00550
  441 FCRMAT('       NO. IN BIN ',I2,' IS',I6)                            DRL00560
      WRITE(6,521)(BINS(I,J,K),K=1,NFLDVW)                               DRL00570
   46 CCNTINUE                                                            DRL00580
      GO TO 48                                                            DRL00590
   47 WRITE(6,521) ((BINS(I,J,K), K = 1,NFLDVW), J = 1,NTHETA)            DRL00600
   48 CONTINUE                                                            DRL00610
      IF(IPRT.LE.2) GO TO 5C                                              DRL00620
      WRITE(6,442)                                                        DRL00630
  442 FCRMAT(/' TIME OF ARRIVAL BINS GIVEN BY',/)                         DRL00640
      DC 49 J = 1,NTHETA                                                  DRL00650
   49 WRITE(6,522) (BINDST(I,J,K), K = 1,20)                              DRL00660
  522 FCRMAT(1X,2CI6)                                                     DRL00670
   5C CONTINUE                                                            DRL00680
      IF(IPRT.GE.6) GO TC 399                                             DRL00690
      IF(IPRT.LT.1) GO TO 1                                               DRL00700
      WRITE(6,450)                                                        DRL00710
  45C FCRMAT(//,' THE NEGATIVE LOG CF RELATIVE FLUX AT EACH',             DRL00720
     *' ANNULAR RING IS')                                                 DRL00730
      DC 60 I = 1,NSHLS                                                   DRL00740
      R = DISTSH*I                                                        DRL00750
      DO 55 J = 1,NTHETA                                                  DRL00760
      TH1 = (PI*(J-1))/NTHETA                                             DRL00770
      IF(METHOD.EC.3) TH1 = (TH1*(J-1))/NTHETA                            DRL00780
      TH2 = (PI*J)/NTHETA                                                 DRL00790
      IF(METHOD.EC.3) TH2 = (TH2*J)/NTHETA                                DRL00800
      AREA = 2.0*PI*R**2*(COS(TH1)-COS(TH2))                              DRL00810
CP    WRITE(7,999) J,AREA                                                 DRL00820
  999 FCRMAT(' FOR SHELL ',I2,' THE AREA IS',F10.6)                       DRL00830
      DC 55 K = 1,NFLDVW                                                  DRL00840
      IF(BINS(I,J,K).EQ.0) RFLUX(I,J,K) = 0.0                             DRL00850
      IF(BINS(I,J,K).EQ.0) GO TO 55                                       DRL00860
      RFLUX(I,J,K) = -ALOG10(BINS(I,J,K)/(AREA*NPHOT))                    DRL00870
   55 CONTINUE                                                            DRL00880
      WRITE(6,452) ((RFLUX(I,J,K), K = 1,NFLDVW),J = 1,NTHETA)            DRL00890
  452 FCRMAT(1X,20F6.2)                                                   DRL00900
   6C CCNTINUE                                                            DRLCC910
  521 FCRMAT(1X,2CI6)                                                     DRL00920
      IF(IPRT.LE.5) GO TC 1                                               DRL00930
  399 DC 400 NN = 1,NSHLS                                                 DRL00940
  400 CALL LSTHG(NTHETA,NN,BINS,METHOD,GIN)                               DRL00950
      GC TO 1                                                             DRLCC960
```

```
      FUNCTION RANEXP(IX,IY,TAU)                                        FUN00010
C**** THIS FUNCTION GENERATES A RANDOM NUMBER WEIGHTED EXPONENTIALLY     FUN00020
      CALL RANDU(IX,IY,RN)                                              FUN00030
      IX = IY                                                           FUN00040
      RANEXP = -TAU*ALOG(1.0-RN)                                        FUN00050
      RETURN                                                            FUN00060
      END                                                               FUN00070
C                                                                       FUN00080
      FUNCTION RANTH(IX,IY,HENA,HENB,HENC,HEND,RPT,FBACK)               FUN00090
C**** THIS FUNCTION GENERATES A RANDOM VALUE OF THETA,                  FUN00100
C       SUITABLY WEIGHTED                                               FUN00110
      DATA PI2/1.5707963268/, PI/3.1415926536/                          FUN00120
      CALL RANDU(IX,IY,RN)                                              FUN00130
      IX = IY                                                           FUN00140
      IF(RN.GT.RPT) GO TO 20                                            FUN00150
      IF(FBACK.EQ.0.0) GO TO 10                                         FUN00160
      CALL RANDU(IX,IY,RN)                                              FUN00170
      IX = IY                                                           FUN00180
      CRANM1 = 0.0                                                      FUN00190
      DO 5 I = 1,8                                                      FUN00200
      CRANTH = HENA-HENB/(RN-HENC-HEND*FBACK*CRANM1*(1.0-CRANM1**2))**2 FUN00210
CP    WRITE(6,999) I,CRANTH,RN                                          FUN00220
  999 FORMAT(1X,I4,2F12.6)                                              FUN00230
      IF(ABS(CRANTH-CRANM1).LT.0.001) GO TO 8                           FUN00240
      CRANM2 = CRANM1                                                   FUN00250
      CRANM1 = CRANTH                                                   FUN00260
    5 CONTINUE                                                          FUN00270
      CRANTH = 0.25*(CRANM2+CRANTH)+0.5*CRANM1                          FUN00280
CP    WRITE(7,998) CRANTH,RN                                            FUN00290
  998 FORMAT(1X,2F10.6)                                                 FUN00300
    8 CONTINUE                                                          FUN00310
      IF(ABS(CRANTH).GT.1.0) CRANTH = CRANTH/(ABS(CRANTH) + .00001)     FUN00320
      RANTH = ARCOS(CRANTH)                                             FUN00330
      RETURN                                                            FUN00340
C**** CALCULATE THETA USING FORWARD SCATTER FUNCTION HERE               FUN00350
   10 CALL RANDU(IX,IY,RN)                                              FUN00360
      IX = IY                                                           FUN00370
C      WRITE(6,990) HENA,HENB,HENC, RN                                  FUN00380
C 990 FORMAT(1X,4F12.6)                                                 FUN00390
      RANTH = ARCOS(HENA-HENB/(HENC-RN)**2)                             FUN00400
      RETURN                                                            FUN00410
C**** CALCULATE THETA ASSUMING RAYLEIGH SCATTERING HERE                 FUN00420
   20 CALL RANDU(IX,IY,RN)                                              FUN00430
      IX = IY                                                           FUN00440
      RLB = 4.0*RN-2.0                                                  FUN00450
      RLBS = SQRT(RLB**2+1.0)                                           FUN00460
      CAPA = (-RLB+RLBS)**(1./3.)                                       FUN00470
      CAPB = -(RLB+RLBS)**(1./3.)                                       FUN00480
```

```
      RANTH = ARCOS(CAPA+CAPB)                                          FUNOC49C
      RETURN                                                            FUN00500
      END                                                               FUN00510
C                                                                       FUN00520
      FUNCTION RANPH(IX,IY)                                             FUN00530
C**** THIS FUNCTION GENERATES A RANDOM VALUE OF PHI                     FUN00540
      DATA TPI/6.283183072/                                             FUN00550
      CALL RANDU(IX,IY,RN)                                              FUN00560
      IX = IY                                                           FUN00570
      RANPH = TPI*RN                                                    FUN00580
      RETURN                                                            FUN00590
      END                                                               FUN00600
```

```
C     THIS SUBROUTINE SIMULATES A PHOTON WHICH RANDOMLY COLLIDES WITH    DRL00010
C     PARTICLES, SCATTERED AT ANGLES WEIGHTED BY VARIOUS FUNCTIONS,      DRL00020
C     AND DETERMINES LOCATION OF INTERSECTION WITH VARIOUS SPHERES       DRL00030
C                                                                        DRL00040
C        INPUT PARAMETERS:                                               DRL00050
C     NPHOT = NUMBER OF PHOTONS TO TRACE THROUGH                         DRL00060
C     NTHETA = THE NUMBER OF THETA "BINS" TO KEEP TRACK OF               DRL00070
C     NFLDVW = THE NUMBER OF "FIELD OF VIEW BINS" FOR EACH THETA BIN     DRL00080
C     NSHLS = THE NUMBER OF SHELLS TO BE INTERSECTED BY EACH PHOTON      DRL00090
C     DISTSH = THE DISTANCE BETWEEN EACH SHELL (AND FROM ORIGIN FOR      DRL00100
C              FIRST)                                                    DRL00110
C     RATIO = RATIO OF SCATTERING CROSS SECTION TO TOTAL EXTINCTION      DRL00120
C     G = G VALUE FOR HENYEY-GREENSTEIN FUNCTION                         DRL00130
C     RPT = RATIO OF PARTICULATE TO TOTAL SCATTERING                     DRL00140
C     FBACK = RATIO OF BACKSCATTER TO TOTAL PARTICULATE SCATTERING       DRL00150
C     IX IS A RANDOM NUMBER TO START THINGS                              DRL00160
C                                                                        DRL00170
C              VARIABLES:                                                DRL00180
C     X(I) = X COORDINATE; I=1 FOR ORIGIN, I=2 FOR X=1, I=3 FOR Z=1      DRL00190
C              (X=1 MEANS (1,0,0) POINT FOR  ORIGINAL COORD SYST, ETC.)  DRL00200
C     Y(I) = Y COORDINATE;  DITTO                                        DRL00210
C     Z(I) = Z    "                "                                     DRL00220
C     XT(I) = THE NEW COORDINATE AFTER ROTATION BY THETA, PHI, AND       DRL00230
C              TRANSLATION BY DISTANCE DR                                DRL00240
C     YT(I), AND ZT(I), LIKEWISE                                         DRL00250
C     XS(I) IS A TEMPORARY COORDINATE POINT,                             DRL00260
C     AT THE INTERSECTION WITH THE SPHERE                                DRL00270
C     YS(I), ZS(I), LIKEWISE                                             DRL00280
C     TH = VALUE OF THETA BY WHICH COORDINATE SYSTEM IS ROTATED          DRL00290
C          AFTER SIMULATED COLLISION WITH PARTICLE                       DRL00300
C     PH = VALUE OF PHI, LIKEWISE                                        DRL00310
C     THP = VALUE OF THETA PRIME, THETA RELATIVE TO PHOTON-FIXED         DRL00320
C           COORDINATE SYSTEM                                            DRL00330
C     PHP = VALUE OF PHI PRIME, IN PHOTON FIXED COORD SYST               DRL00340
C     BINS(I,J,K) = NO OF OCCURRENCES AT ITH SPHERE WITH JTH             DRL00350
C              BIN OF THETA AND KTH BIN OF THETA PRIME                   DRL00360
C     RSHL(I) = DISTANCE OF ITH SHELL FROM ORIGIN                        DRL00370
C     DTHETA = NEW VALUE OF THETA RELATIVE TO OLD DIRECTION              DRL00380
C     DPHI = NEW VALUE OF PHI RELATIVE TO OLD DIRECTION                  DRL00390
C     DR = DISTANCE OF PHOTON TRAVEL AFTER A COLLISION                   DRL00400
C     IX, IY = FIXED-POINT RANDOM NUMBERS                                DRL00410
C     RTOT = TOTAL DISTANCE A PHOTON TRAVELS                             DRL00420
C     DIST = DISTANCE FROM ORIGIN OF PHOTON                              DRL00430
C     SDIST = DISTANCE FROM ORIGIN FOR PREVIOUS CALCULATION              DRL00440
C     PDIST = DISTANCE ACTUALLY TRAVELED FROM ORIGIN TO SHELL            DRL00450
C     ISAV =  INTEGER CORRESPONDING TO NUMBER OF SHELL                   DRL00460
C             BEYOND WHICH THE PHOTON IS LOCATED                         DRL00470
C     XPDR =  RATIO OF DISTANCE FROM POINT OF COLLISION TO A             DRL00480
```

```
C             SHELL RELATIVE TO TOTAL DISTANCE A PHOTON                  DRL00490
C             TRAVELS AFTER A COLLISION                                  DRL00500
      SUBROUTINE LITE(NPHOT,NTHETA,NFLDVW,NSHLS,DISTSH,RATIO,            DRL00510
     *G,RPT,FBACK,IX,BINS,BINDST,METHOD,IPRT)                            DRL00520
      INTEGER*4 BINS(20,50,1), BINDST(20,50,20)                          DRL00530
      DIMENSION X(4), Y(4), Z(4), XT(4), YT(4), ZT(4), XS(4),            DRL00540
     * YS(4), ZS(4), RSHL(20), VX(3), VY(3), VZ(3)                       DRL00550
      DATA PI/3.1415926536/, TPI/6.2831853072/, RCOTPI/1.772453851/      DRL00560
      DATA EPSIL/1.00E-03/                                               DRL00570
      WRITE(6,888)                                                       DRL00580
  888 FORMAT(' ENTERED LITE')                                            DRL00590
      WRITE(6,980) NPHOT,NTHETA,NFLDVW,NSHLS,IPRT,METHOD,DISTSH,         DRL00600
     * RATIO,G,RPT,FBACK,IX                                             DRL00610
  980 FORMAT(1H1,' NPHOT = ',I5,'   NTHETA = ',I3,'   NFLDVW = ',I3,     DRL00620
     *'   NSHLS = ',I2,/,'   METHOD = ',2I1,'   DISTSH = ',F5.3,        DRL00630
     *'   RATIO = ',F6.4,'   G = ',F7.5,'   RPT = ',F7.5,               DRL00640
     *'   FBACK = ',F7.5,'   IX = ',I10)                                 DRL00650
      HENA = (1.0+G**2)/(2.0*G)                                          DRL00660
      HENB = (1.0-G**2)**2/(8.0*G**3)                                    DRL00670
      HENC = (G+1.0)/(2.0*G)                                             DRL00680
      HEND = (1.0-G**2)/(4.0*(1.0+G**2)**1.5)                            DRL00690
C**** INITIALIZE ARRAYS                                                  DRL00700
      DO 5 I = 1,NSHLS                                                   DRL00710
      DO 5 J = 1,NTHETA                                                  DRL00720
      DO 4 K = 1,20                                                      DRL00730
    4 BINDST(I,J,K) = 0                                                  DRL00740
      DO 6 K = 1,NFLDVW                                                  DRL00750
    6 BINS(I,J,K) = 0                                                    DRL00760
    5 CONTINUE                                                           DRL00770
      D = -DISTSH                                                        DRL00780
      NUP = NSHLS+1                                                      DRL00790
      DO 10 I = 1,NUP                                                    DRL00800
      D = D+DISTSH                                                       DRL00810
   10 RSHL(I) = D                                                        DRL00820
C**** START LOOP FOR EACH PHOTON                                         DRL00830
      DO 100 NPH = 1,NPHOT                                               DRL00840
      IXS = IX                                                           DRL00850
      NSCA = 0                                                           DRL00860
      RTOT = 0.0                                                         DRL00870
      ISAV = 1                                                           DRL00880
      DO 15 I = 1,4                                                      DRL00890
      X(I) = 0.0                                                         DRL00900
      Y(I) = 0.0                                                         DRL00910
   15 Z(I) = 0.0                                                         DRL00920
      X(2) = 1.0                                                         DRL00930
      Y(4) = 1.0                                                         DRL00940
      Z(3) = 1.0                                                         DRL00950
C**** CHOOSE DISTANCE OF INITIAL PHOTON                                  DRL00960
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```
      DR = RANEXP(IX,IY,1.0)                                          DRLCC97C
      DC 17 I = 1,4                                                   DRLC0980
   17 Z(I) = Z(I) + DR                                                DRLCC990
   18 IF(DR.LT.RSHL(ISAV+1)) GO TC 19                                 DRL01000
      ISAV = ISAV+1                                                   DRL01010
      IF(ISAV.GT.NSHLS) GC TO 100                                     DRL01020
      GC TO 18                                                        DRL01030
   19 CONTINUE                                                        DRL01040
      RTOT = DR                                                       DRL01050
      SCIST = RTOT                                                    DRLC1060
C**** CETERMINE IF PHOTCN IS ABSORBED CR SCATTERED                    DRL01070
   2C CALL RANDU(IX,IY,RN)                                            DRL01080
      IX = IY                                                         DRL01090
      IF(RN.GT.RATIO) GC TO 100                                       DRL01100
C**** IF PHOTON IS SCATTERED, DETERMINE NEW ANGLES AND DISTANCES      DRL01110
      NSCA = NSCA+1                                                   DRL01120
      DR = RANEXP(IX,IY,1.0)                                          DRL01130
      DTHETA = RANTH(IX,IY,HENA,HENB,HENC,HEND,RPT,FBACK)             DRL01140
      DPHI = RANPH(IX,IY)                                             DRL01150
      STH = SIN(DTHETA)                                               DRL01160
      CTH = CCS(DTHETA)                                               DRL01170
      SPH = SIN(DPHI)                                                 DRL01180
      CPH = CCS(DPHI)                                                 DRL01190
CP    WRITE(6,990) RTOT,DR,DTHETA,DPHI,STH,CTH,SPH,CPH                DRL01200
  990 FCRMAT(/' RTOT, DR, DTHETA, DPHI, STH, CTH, SPH, CPH:',/,       DRL01210
     * 1X,8F10.5)                                                     DRL01220
C**** RCTATE TO NEW PHOTCN POSITION ANC TRANSLATE                     DRL01230
      DC 30 I = 1,4                                                   DRL01240
      XT(I) = X(I)*CTH*CPH-Y(I)*CTH*SPH-Z(I)*STH                      DRL01250
      YT(I) = X(I)*SPH + Y(I)*CPH                                     DRL01260
      ZT(I) = X(I)*STH*CPH - Y(I)*STH*SPH + Z(I)*CTH + DR             DRLC1270
CP    WRITE(6,991) XT(I), YT(I), ZT(I)                                DRL01280
  991 FCRMAT(/' CCORDINATES', 3F10.6)                                 DRL01290
   3C CCNTINUE                                                        DRL01300
      DIST = SQRT(XT(1)**2+YT(1)**2+ZT(1)**2)                         DRL01310
C**** CHECK TO SEE IF PHCTCN HAS PENETRATED A SHELL                   DRL01320
C**** CCNSIDER IF PHOTON HAS PENETRATED IN AND OUT CF SHELL           DRL01330
C     FF IS FRACTIONAL CISTANCE BETWEEN TWO POINTS GIVING             DRL01340
C        THE DISTANCE OF CLOSEST APPRCACH                             DRL01350
      FF = 1.0-ZT(1)/DR                                               DRL01360
      IF((FF.LT.0.0).OR.(FF.GT.1.0)) GC TO 35                         DRL01370
C     DC IS DISTANCE OF CLOSEST APPROACH                              DRL01380
      DC = SQRT(XT(1)**2+YT(1)**2)                                    DRL01390
      IF(DC.GT.RSHL(ISAV)) GO TO 35                                   DRL01400
   34 ISAV = ISAV-1                                                   DRL01410
CP    WRITE(7,920) DC,RSHL(ISAV),FF,IX                                DRL01420
  920 FCRMAT(1X,3F10.5,1X,I10)                                        DRL01430
      IF(DC.GT.RSHL(ISAV)) GO TO 50                                   DRL01440
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      GC TO 35                                                          DRL01450
C**** CCNSIDER IF PHOTON HAS PENETRATED CUTER SHELLS                    DRL01460
   35 IF((ISAV).GT.(NSHLS)) GO TO 100                                   DRL01470
   36 IF(DIST.GT.RSHL(ISAV+1)) GO TO 50                                 DRL01480
   37 IF(DIST.GT.RSHL(ISAV)) GO TO 40                                   DRL01490
      ISAV = ISAV-1                                                     DRL01500
      GO TO 37                                                          DRL01510
C**** TIME TO UPDATE COORDINATES - GET READY FOR ANCTHER CCLLISICN      DRL01520
   40 RTCT = RTCT+DR                                                    DRL01530
      SDIST = DIST                                                      DRL01540
      DC 45 I = 1,4                                                     DRL01550
      X(I) = XT(I)                                                      DRL01560
      Y(I) = YT(I)                                                      DRL01570
      Z(I) = ZT(I)                                                      DRL01580
   45 CONTINUE                                                          DRL01590
C**** GO TO 20 FOR ANOTHER COLLISION                                    DRL01600
      GC TO 20                                                          DRL01610
C**** CALCULATE CCORDINATES OF POINT AT WHICH PHOTCN PENETRATES SHELL   DRL01620
   50 ISAV = ISAV+1                                                     DRL01630
CP    WRITE(6,551) DR,ZT(1)                                            DRL01640
  551 FCRMAT('**** DR,ZT(1) **',2F10.5)                                 DRL01650
CP    WRITE(6,552) SDIST,RSHL(ISAV)                                    DRL01660
  552 FCRMAT('**** SDIST,RSHL(ISAV)  **',2F10.5)                        DRL01670
CP    WRITE(6,553) DIST,RTOT,ISAV                                      DRL01680
  553 FCRMAT('**** DIST,RTOT,ISAV **',2F10.5,2X,I2)                     DRL01690
      XPDR = (DR-ZT(1)+SQRT((ZT(1)-DR)**2-SDIST**2+RSHL(ISAV)**2))/DR   DRL01700
CP    WRITE(6,550) XPDR                                                DRL01710
  550 FCRMAT(' XPDR = ',F10.5)                                          DRL01720
      DC 55 I = 1,4                                                     DRL01730
      XS(I) = XT(I)                                                     DRL01740
      YS(I) = YT(I)                                                     DRL01750
      ZS(I) = ZT(I) - (1.0-XPDR)*DR                                     DRL01760
   55 CONTINUE                                                          DRL01770
      PDIST = RTOT+XPDR*DR                                              DRL01780
CP    WRITE(6,992) (XS(I),YS(I),ZS(I), I = 1,4)                        DRL01790
  992 FCRMAT(' COORD. AT SHELL: ',12F9.5)                               DRL01800
C**** CALCULATE VALUE OF THETA                                          DRL01810
      VAL = ((XS(1)*(XS(1)-XS(3)) + YS(1)*(YS(1)-YS(3))                 DRL01820
     * + ZS(1)*(ZS(1)-ZS(3)))/RSHL(ISAV))                               DRL01830
      IF(ABS(VAL).GT.1.1) CALL PERR(1,IXS,VAL,XS,YS,ZS)                 DRL01840
      IF(ABS(VAL).GT.1.1) WRITE(6,200) VAL, NPH, IXS                    DRL01850
  200 FCRMAT(' ARCOS GT. 1.1; VAL = ',F10.5,'  NPHCT = ',I5,'   IX = ', DRL01860
     * I10)                                                             DRL01870
      IF(VAL.GT.1.0) VAL = 1.0                                          DRL01880
      IF(VAL.LT.-1.0) VAL = -1.0                                        DRL01890
      TH = PI - ARCOS(VAL)                                              DRL01900
      IF(TH.LT.0.C) TH = 0.0                                            DRL01910
CP    WRITE(6,999) RSHL(ISAV)                                          DRL01920
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  995 FORMAT(' AAA', F10.5)                                                    DRL01930
C**** CALCULATE VALUE OF PHI                                                   DRL01940
      DENOMS = (RSHL(ISAV)**2 - (XS(1)*(XS(3)-XS(1)) + YS(1)*                  DRL01950
     * (YS(3)-YS(1)) + ZS(1)*(ZS(3)-ZS(1)))**2)                                DRL01960
      IF(DENOMS.LT.-EPSIL) CALL PERR(5,IXS,DENOMS,XS,YS,ZS)                    DRL01970
      IF(DENOMS.LT.EPSIL) DENOMS = 0.0                                         DRL01980
      DENOM = SQRT(DENOMS)                                                     DRL01990
CP    WRITE(6,996) DENOM, IXS                                                  DRL02000
      IF(DENOM.EQ.0.0) PH = 0.0                                                DRL02010
      IF(DENOM.EQ.0.0) GO TO 158                                               DRL02020
  996 FORMAT(' DENOM: ',F10.6,I10)                                             DRL02030
      VAL = ((XS(1)*(XS(1)-XS(2)) + YS(1)*(YS(1)-YS(2)) +                      DRL02040
     * ZS(1)*(ZS(1)-ZS(2)))/DENOM)                                             DRL02050
      IF(ABS(VAL).GT.1.1) CALL PERR(2,IXS,VAL,XS,YS,ZS)                        DRL02060
      IF(ABS(VAL).GT.1.1) WRITE(6,200) VAL, NPH, IXS                           DRL02070
      IF(VAL.GT.1.0) VAL = 1.0                                                 DRL02080
      IF(VAL.LT.-1.0) VAL = -1.0                                               DRL02090
      PH = ARCOS(VAL)                                                          DRL02100
      IF((XS(1)*(XS(1)-XS(4))+YS(1)*(YS(1)-YS(4))+ZS(1)*(ZS(1)-ZS(4))          DRL02110
     *).LT.0.0) PH = TPI-PH                                                    DRL02120
  158 CONTINUE                                                                 DRL02130
CP    WRITE(6,998)                                                             DRL02140
  998 FORMAT(' BBB')                                                           DRL02150
C**** CALCULATE VALUE OF THETA PRIME (RECEIVER)                                DRL02160
      VAL = (ZS(1)/RSHL(ISAV))                                                 DRL02170
      IF(ABS(VAL).GT.1.1) CALL PERR(3,IXS,VAL,XS,YS,ZS)                        DRL02180
      IF(ABS(VAL).GT.1.1) WRITE(6,200) VAL, NPH, IXS                           DRL02190
      IF(VAL.GT.1.0) VAL = 1.0                                                 DRL02200
      IF(VAL.LT.-1.0) VAL = -1.0                                               DRL02210
      THP = ARCOS(VAL)                                                         DRL02220
CP    WRITE(6,997)                                                             DRL02230
  997 FORMAT(' CCC')                                                           DRL02240
C**** CALCULATE VALUE OF PHI PRIME (RECEIVER)                                  DRL02250
      PARAM = (XS(1)*XS(3) + YS(1)*YS(3) + ZS(1)*ZS(3))/                       DRL02260
     * (RSHL(ISAV)*SQRT(XS(3)**2 + YS(3)**2 + ZS(3)**2))                        DRL02270
CP    WRITE(6,911) IXS, PARAM                                                  DRL02280
  911 FORMAT(' IXS, PARAM = ',I10,E12.6)                                       DRL02290
      IF(PARAM.LT.0.999) GO TO 159                                             DRL02300
      VAL = XS(2)-XS(1)                                                        DRL02310
      GO TO 585                                                                DRL02320
  159 BP = -RSHL(ISAV)**2/(XS(3)*XS(1)+YS(3)*YS(1)+ZS(3)*ZS(1))                DRL02330
      VX(1) = XS(1)+BP*XS(3)                                                   DRL02340
      VY(1) = YS(1)+BP*YS(3)                                                   DRL02350
      VZ(1) = ZS(1)+BP*ZS(3)                                                   DRL02360
      ANORM = SQRT(VX(1)**2+VY(1)**2+VZ(1)**2)                                 DRL02370
      VX(1) = VX(1)/ANORM                                                      DRL02380
      VY(1) = VY(1)/ANORM                                                      DRL02390
      VZ(1) = VZ(1)/ANORM                                                      DRL02400
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      VX(3) = XS(1)/RSHL(ISAV)                                          DRL02410
      VY(3) = YS(1)/RSHL(ISAV)                                          DRL02420
      VZ(3) = ZS(1)/RSHL(ISAV)                                          DRL02430
      VX(2) = VY(3)*VZ(1)-VZ(3)*VY(1)                                   DRL02440
      VY(2) = VZ(3)*VX(1)-VX(3)*VZ(1)                                   DRL02450
      VZ(2) = VX(3)*VY(1)-VY(3)*VX(1)                                   DRL02460
CP    WRITE(6,989) BP,ANORM,((VX(I),VY(I),VZ(I)),I=1,3)                 DRL02470
  989 FORMAT(' BP, ANORM, V(I), V(I), V(I): ',11E8.4)                   DRL02480
      IF(VZ(3).LT.0.9999) GO TO 58                                      DRL02490
      VAL = 1.0                                                         DRL02500
      GO TO 59                                                          DRL02510
   58 CONTINUE                                                          DRL02520
      VAL = VZ(1)/SQRT(1.0-VZ(3)**2)                                    DRL02530
  585 IF(ABS(VAL).GT.1.1) CALL PERR(4,3,RSHL(ISAV),VX,VY,VZ)            DRL02540
      IF(ABS(VAL).GT.1.1) CALL PERR(4,IXS,VAL,XS,YS,ZS)                 DRL02550
      IF(ABS(VAL).GT.1.1) WRITE(6,200) VAL, NPH, IXS                    DRL02560
      IF(VAL.GT.1.0) VAL = 1.0                                          DRL02570
      IF(VAL.LT.-1.0) VAL = -1.0                                        DRL02580
   59 PHP = ARCOS(VAL)                                                  DRL02590
      IF(VZ(2).LT.0.0) PHP = TPI-PHP                                    DRL02600
CP    WRITE(6,995)                                                      DRL02610
  995 FORMAT(' DDD')                                                    DRL02620
C**** TALLY LOCATION OF ANGULAR RESULTS                                 DRL02630
      IF(METHOD.LE.2) NTH = (TH*NTHETA/PI) + 1                          DRL02640
      IF(METHOD.GE.3) NTH = (SQRT(TH)*NTHETA)/ROOTPI + 1                DRL02650
      IF(METHOD.LE.2) NTHP = 2.0*THP*NFLDVW/PI + 1                      DRL02660
      IF(METHOD.GE.3) NTHP = (SQRT(2.0*THP)*NFLDVW)/ROOTPI + 1          DRL02670
      IF(IPRT.NE.9) GO TO 70                                            DRL02680
CP    WRITE(9,993)NPH,ISAV,NTH,NSCA,TH,PH,THP,PHP,PDIST                 DRL02690
  993 FORMAT(4I5,4F10.5,F10.5)                                          DRL02700
   70 CONTINUE                                                          DRL02710
      BINS(ISAV-1,NTH,NTHP) = BINS(ISAV-1,NTH,NTHP) + 1                 DRL02720
CX    XXXX = ((YS(3)-YS(1))*ZS(1) - (ZS(3)-ZS(1))*YS(1))*VX(1)          DRL02730
CX    YYXX = ((ZS(3)-ZS(1))*XS(1) - (XS(3)-XS(1))*ZS(1))*VY(1)          DRL02740
CX    ZZXX = ((XS(3)-XS(1))*YS(1) - (YS(3)-YS(1))*XS(1))*VZ(1)          DRL02750
CP    WRITE(6,986)XXXX,YYXX,ZZXX                                        DRL02760
  986 FORMAT(' XXXX, YYXX, ZZXX :',3F10.6)                              DRL02770
C**** TALLY LOCATION OF INTERSECTION IN DISTANCE OF ARRIVAL BINSA        DRL02780
      IF(PDIST.LE.(1.00001*RSHL(ISAV))) PDIST = 1.00001*RSHL(ISAV)      DRL02790
      XDIST = ALOG10((PDIST/RSHL(ISAV))-1.0) + 4.0                      DRL02800
      IF(XDIST.LT.0.0) XDIST = 0.0                                      DRL02810
      IF(XDIST.GE.4.99) XDIST = 4.99                                    DRL02820
      NDIST = (XDIST*20)/5 + 1                                          DRL02830
      BINDST(ISAV-1,NTH,NDIST) = BINDST(ISAV-1,NTH,NDIST) + 1           DRL02840
      GO TO 35                                                          DRL02850
  100 CONTINUE                                                          DRL02860
      RETURN                                                            DRL02870
      END                                                               DRL02880
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C   WRITE FUNCTION SUBPROGRAM TO CALCULATE THE FUNCTION TO BE LEAST SQUARLST00010
C   A IS DIMENSIONED 10 AND CONTAINS THE PARAMETERS TO BE VARIED SUCH AS LST00020
C   MINIMIZE THE SUM OF ERRORS SQUARED X, XB, AND XC ARE THE THREE INDEPELSTC0030
C   PARAMETERS (YOU MAY WISH TO USE ONLY X) NOF IS THE FUNCTION NUMBER - LST00040
C   BRANCHING TO DIFFERENT PARTS OF THE EQN SUBPROGRAM WHEN SEVERAL FUNCTLST00050
C   ARE TO BE FIT (FOR SEVERAL JOBS TO BE DONE).                        LST00060
C   THE DIMENSIONED ARRAYS HAVE THE FOLLOWING MEANING--                 LST00070
C      A(10) CONTAINS THE PARAMETERS WHICH ARE TO BE VARIED             LST00080
C   X(200), XB(200), AND XC(200) CONTAIN THE INDEPENDENT PARAMETERS.     LST00090
C   Z(200) CONTAINS THE OBSERVED VALUES OF THE FUNCTION (THE OBSERVED DEPLST00100
C   PARAMETERS).                                                        LST00110
C   FINCR(10) CONTAINS THE MAGNITUDE OF THE INCREMENTS FOR THE PARAMETERSLST00120
C   SO THAT THE PROGRAM MAY TAKE NUMERICAL DERIVATIVES WITH REASONABLE ACLST00130
C   R(200) CONTAINS THE DIFFERENCE BETWEEN OBSERVED (Z(200)) AND CALCULATLSTC0140
C   E(10) CONTAINS THE ESTIMATES OF THE ERRORS OF THE PARAMETERS A(10) AFLST00150
C   ITERATION.                                                          LST00160
C      INPUT                                                            LST00170
C   FIRST CARD - WILL BE REPRODUCED ON OUTPUT USED FOR LABELING.        LST00180
C   SECOND CARD FORMAT(I2,I3,I2,I3,E10.2)                               LST00190
C   IR= NUMBER OF PARAMETERS TO BE VARIED                               LST00200
C   IS= NUMBER OF POINTS                                                LST00210
C   NOF = FUNCTION NUMBER (SEE ABOVE)                                   LST00220
C   NINP = NUMBER OF INDEPENDENT PARAMETERS                             LST00230
C   EPSIL = IS USED AS A CRITERION FOR CONVERGENCE. IF THE RELATIVE VALUELST00240
C   OF THE RESIDUAL CHANGES BY LESS THAN EPSIN IN TWO SUCCESSIVE ITERATIOLST00250
C   CONVERGENCE IS REACHED.                                             LST00260
      SUBROUTINE LSTHG(IS,NN,IBIN,NOF,XPNIN)                            LST00270
      IMPLICIT REAL*8 (A-H,O-Z)                                         LST00280
      INTEGER*4 IBIN(20,50,1)                                           LST00290
      DIMENSION A(10), X(200), XB(200), XC(200), Z(200), FINCR(10),     LSTC0300
     1R(200), E(10)                                                     LST00310
      CALL ERRSET(208,0,-1,0,0,0)                                       LST00320
      DO 2 I = 1,IS                                                     LST00330
    2 Z(I) = IBIN(NN,I,1)                                               LST00340
      SUM = 0.0                                                         LST00350
      DO 5 I = 1,IS                                                     LST00360
    5 SUM = SUM+Z(I)                                                    LST00370
      IF(SUM.LT.20) RETURN                                              LST00380
      IR = 2                                                            LST00390
      NINP = 1                                                          LST00400
      EPSIL = .0001                                                     LST00410
      GO TO (7,8,8), NOF                                                LST00420
    7 CONTINUE                                                          LST00430
      A(1) = SUM                                                        LST00440
      A(2) = .90                                                        LST00450
      A(10) = 180./IS                                                   LST00460
      GO TO 9                                                           LST00470
    8 A(1) = .8                                                         LST00480
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      A(2) = XPNIN                                                      LSTCC490
      IF(A(2).EQ.C.0) A(2) = 1.05                                       LSTCC500
      A(9) = SUM                                                        LSTCC510
      A(10) = 180./IS                                                   LSTCC520
    9 CCNTINUE                                                          LSTCC530
      FINCR(1) = .001                                                   LSTCC540
      FINCR(2) = .001                                                   LSTCC550
      FINCR(3) = .001                                                   LSTCC560
      FINCR(4) = .001                                                   LSTCC570
      GO TO (10,1C,12), NOF                                             LSTCC580
   10 DEL = 180./IS                                                     LSTCC590
      ANGLE = -DEL                                                      LSTCC600
      DO 11 I = 1,IS                                                    LSTCC610
      ANGLE = ANGLE+DEL                                                 LSTCC620
   11 X(I) = ANGLE                                                      LSTCC630
      GO TO 15                                                          LSTCC640
   12 IS1 = IS + 1                                                      LSTCC650
      DO 13 I = 1,IS1                                                   LSTCC660
      FNUM = I-1                                                        LSTCC670
      DENOM = IS                                                        LSTCC680
   13 X(I) = 180.C*(FNUM/DENOM)**2                                      LSTCC690
   15 CCNTINUE                                                          LSTCC700
   20 WRITE(6,106)                                                      LSTCC710
      WRITE(6,109)                                                      LSTCC720
      WRITE(6,107)                                                      LSTCC730
      WRITE(6,108) (IR,IS,NOF,NINP,EPSIL)                               LSTCC740
      WRITE(6,109)                                                      LSTCC750
      WRITE(6,110)                                                      LSTCC760
      WRITE(6,109)                                                      LSTCC770
      WRITE(6,111) (A(I), I = 1,IR)                                     LSTCC780
      WRITE(6,109)                                                      LSTCC790
      WRITE(6,112)                                                      LSTCC800
      WRITE(6,109)                                                      LSTCC810
      WRITE(6,111) (FINCR(I), I = 1,IR)                                 LSTCC820
      WRITE(6,109)                                                      LSTCC830
      WRITE(6,113)                                                      LSTCC840
      WRITE(6,109)                                                      LSTCC850
      WRITE(6,114) (Z(I), I = 1,IS)                                     LSTCC860
      WRITE(6,109)                                                      LSTCC870
      WRITE(6,115)                                                      LSTCC880
      WRITE(6,109)                                                      LSTCC890
      WRITE(6,116) (X(I), I = 1,IS)                                     LSTCC900
      WRITE(6,109)                                                      LSTCC910
      IF(NINP-2) 30,25,25                                               LSTCC920
   25 WRITE(6,117) (XB(I), I = 1,IS)                                    LSTCC930
      WRITE(6,109)                                                      LSTCC940
      IF(NINP-3) 30,26,26                                               LSTCC950
   26 WRITE(6,118) (XC(I), I = 1,IS)                                    LSTCC960
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```
      WRITE(6,109)                                                      LST00970
C  30 WRITE(6,999)                                                      LST00980
 999 FORMAT(' REACHED LEAST')                                           LST00990
  30 CALL LEAST(IR,IS,A,X,XB,XC,Z,FINCR,EPSIL,NOITR,RRQ,NOF,R,E)        LST01000
      WRITE(6,109)                                                      LST01010
      WRITE(6,119)                                                      LST01020
      WRITE(6,109)                                                      LST01030
      WRITE(6,120) (A(I), I = 1,IR)                                     LST01040
      WRITE(6,109)                                                      LST01050
      WRITE(6,124)                                                      LST01060
      WRITE(6,109)                                                      LST01070
      WRITE(6,120) (E(I), I = 1,IR)                                     LST01080
      WRITE(6,109)                                                      LST01090
      WRITE(6,121) NOITR, RRQ                                           LST01100
      WRITE(6,109)                                                      LST01110
      WRITE(6,122)                                                      LST01120
      WRITE(6,109)                                                      LST01130
      WRITE(6,123) (R(I), I = 1,IS)                                     LST01140
      RETURN                                                            LST01150
 100 FORMAT (I2,I3,I2,I3,E10.2)                                         LST01160
 101 FORMAT(16F5.0)                                                     LST01170
 102 FORMAT (16F5.0)                                                    LST01180
 103 FORMAT (16F5.0)                                                    LST01190
 104 FORMAT (16F5.0)                                                    LST01200
 105 FORMAT (16F5.0)                                                    LST01210
 106 FORMAT (80H                                                        LST01220
     1                    )                                           LST01230
 107 FORMAT (99H NUMBER OF PARAMETERS     NUMBER OF POINTS     FUNCTIONLST01240
     1 NUMBER     NUMBER OF INDEP PARAM     EPSILON)                  LST01250
 108 FORMAT (8X,I2,24X,I3,17X,I2,19X,I2,12X,E10.2)                      LST01260
 109 FORMAT (1H )                                                       LST01270
 110 FORMAT (26H THE PARAMETERS FED IN ARE)                             LST01280
 111 FORMAT (1H ,10F10.5)                                               LST01290
 112 FORMAT (55H THE INCREMENTS FOR OBTAINING NUMERICAL DERIVATIVES ARELST01300
     1)                                                                 LST01310
 113 FORMAT (34H THE OBSERVED VALUES TO BE FIT ARE)                     LST01320
 114 FORMAT (1X,10F10.2)                                                LST01330
 115 FORMAT (31H THE INDEPENDENT PARAMETERS ARE)                        LST01340
 116 FORMAT (1H ,10F10.5)                                               LST01350
 117 FORMAT (1H ,10F10.5)                                               LST01360
 118 FORMAT (1H ,10F10.5)                                               LST01370
 119 FORMAT (38H THE BEST VALUES OF THE PARAMETERS ARE)                 LST01380
 120 FORMAT(1H ,7F12.5)                                                 LST01390
 121 FORMAT (24H NUMBER OF ITERATIONS = ,I2,50H    THE SUM OF THE SQUARLST01400
     1ES OF THE ERRORS IS NOW = ,E12.6)                                 LST01410
 122 FORMAT (53H OBSERVED MINUS CALCULATED VALUES OF THE BEST FIT ARE) LST01420
 123 FORMAT (1H ,10F10.2)                                               LST01430
 124 FORMAT (66H ESTIMATES OF THE ERROR IN EACH PARAMETER ARE (STANDARDLST01440
```

```
     1 DEVIATION))                                                          LST01450
  200 STOP                                                                  LST01460
      END                                                                   LST01470
C                                                                           LST01480
      SUBROUTINE LEAST(IR,IS,A,X,XB,XC,Z,FINCR,EPSIL,NOITR,RRQ,NOF,R,E)     LST01490
      IMPLICIT REAL*8 (A-H,O-Z)                                             LST01500
C   IR = NO. OF PARAMETERS, IS = NO. OF POINTS, A IS ARRAY OF PARAMETERS,LST01510
C   X IS INDEPENDENT VARIABLE( Z DEPENDENT.  FINCR IS ARRAY OF INCREMENTSLST01520
C PARAMETERS.  EPSIL IS  -FRACTIONAL-  ERROR CRITERION.  NOITR IS NO. OFLST01530
C ITERATIONS REQUIRED (UP TO 10).  RRQ = SUM OF SQUARES OF RESIDUALS)   LST01540
C   NOF IS NUMBER OF THE FUNCTION USED IN -EQN-.                          LST01550
C E  IS THE ARRAY OF ESTIMATED ERRORS IN THE PARAMETERS                  LST01560
      DIMENSION A(10), X(200), XB(200), XC(200), Z(200), FINCR(10), R(20LST01570
     10), D(200,10), DT(10,200), DEL(10), DS(10), DPI(10,10), DP(10,10),LST01580
     2E(10)                                                             LST01590
      NOITR = 0                                                         LST01600
      DO 20 I = 1,IS                                                    LST01610
   20 R(I) = Z(I) - EQN(A,X(I),X(I+1),XC(I),NOF)                        LST01620
   25 IF (NOITR-9) 130,130,4                                            LST01630
  130 NOITR = NOITR + 1                                                 LST01640
      DO 220 J = 1, IR                                                  LST01650
      A(J) = A(J)+FINCR(J)                                              LST01660
      DO 15 I = 1,IS                                                    LST01670
   15 D(I,J) = EQN(A,X(I),X(I+1),XC(I),NOF)                             LST01680
      A(J) = A(J)-FINCR(J)                                              LST01690
  220 CONTINUE                                                          LST01700
      DO 30 I = 1,IS                                                    LST01710
      CONST = EQN(A,X(I),X(I+1),XC(I),NOF)                              LST01720
      DO 30 J = 1,IR                                                    LST01730
   30 D(I,J) = (D(I,J)-CONST)/FINCR(J)                                  LST01740
      DO 35 I = 1,IS                                                    LST01750
      DO 35 J = 1,IR                                                    LST01760
   35 DT(J,I) = D(I,J)                                                  LST01770
C     DO 998 I = 1,IR                                                   LST01780
C 998 WRITE(6,999) (DT(I,J), J = 1,IS)                                  LST01790
  999 FORMAT(1X,12F10.5)                                                LST01800
      DO 36 I = 1,IR                                                    LST01810
      DO 36 J = 1,IR                                                    LST01820
      DP(I,J) = 0.0                                                     LST01830
      DO 36 K = 1,IS                                                    LST01840
   36 DP(I,J) = DP(I,J)+DT(I,K)*D(K,J)                                  LST01850
      CALL GAUSS3 (IR,1.00E-30,DP,DPI,KER)                              LST01860
      IF(KER-1) 120,37,120                                              LST01870
   37 DO 40 I = 1,IS                                                    LST01880
      DO 38 J = 1,IR                                                    LST01890
      DS(J) = 0.0                                                       LST01900
      DO 38 K = 1,IR                                                    LST01910
   38 DS(J) = DS(J)+DPI(J,K)*DT(K,I)                                    LST01920
```

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      DO 39 L = 1,IR                                                    LST01930
   39 DT(L,I) = DS(L)                                                   LST01940
   40 CONTINUE                                                          LST01950
C     DO 993 I = 1,IR                                                   LST01960
C 993 WRITE(6,992) (DPI(I,J), J = 1,IR)                                 LST01970
  992 FORMAT(1X,12F10.5)                                                LST01980
C     DO 990 I = 1,IR                                                   LST01990
C 990 WRITE(6,991) (DT(I,J), J = 1,IS)                                  LST02000
  991 FORMAT(1X,1CE12.4)                                                LST02010
C     WRITE(6,991) (R(I), I = 1,IS)                                     LST02020
      DO 110 I = 1, IR                                                  LST02030
      DEL(I) = 0.0                                                      LST02040
      DO 110 J = 1, IS                                                  LST02050
  110 DEL(I) = DEL(I)+DT(I,J)*R(J)                                      LST02060
      DO 10 I = 1,IR                                                    LST02070
   10 A(I) = A(I)+DEL(I)                                                LST02080
      IF(NOF.GE.4) GO TO 222                                            LST02090
      IF(NOF.GE.2) GO TO 219                                            LST02100
      IF(A(2).GT..99) A(2) = 0.99                                       LST02110
      GO TO 222                                                         LST02120
  219 IF(A(1).GT..99) A(1) = 0.99                                       LST02130
      IF(A(2).LT.0.3) A(2) = 0.3                                        LST02140
  222 CONTINUE                                                          LST02150
      DO320 I = 1,IS                                                    LST02160
  320 R(I) = Z(I) - EQN(A,X(I),X(I+1),XC(I),NOF)                        LST02170
      RRQ = 0.0                                                         LST02180
      DO 50 I = 1,IS                                                    LST02190
   50 RRQ = RRQ+R(I)**2                                                 LST02200
      WRITE(6,102) (A(I), I = 1,IR), RRQ                                LST02210
  102 FORMAT(1H ,10E12.4)                                               LST02220
      CRES = DABS(RRQ-RRP) - EPSIL*RRP                                  LST02230
      RRP = RRQ                                                         LST02240
      IF(CRES) 100,100,25                                               LST02250
  101 FORMAT (20H CONVERGENCE FAILURE)                                  LST02260
    4 WRITE(6,101)                                                      LST02270
      GO TO 100                                                         LST02280
  120 WRITE(6,1001)                                                     LST02290
 1001 FORMAT (16H MATRIX SINGULAR)                                      LST02300
  100 FIS1 = IS-IR                                                      LST02310
      DO 150 I = 1,IR                                                   LST02320
  150 E(I) = DSQRT(RRQ*CPI(I,I))                                        LST02330
      RETURN                                                            LST02340
      END                                                               LST02350
C                                                                       LST02360
      SUBROUTINE   GAUSS3(N,EP,A,X,KER)                                 LST02370
      IMPLICIT REAL*8 (A-H,C-Z)                                         LST02380
      DIMENSION A(10,10), X(10,10)                                      LST02390
      DO 1 I=1,N                                                        LST02400
```

```
      DO 1 J=1,N                                                        LST02410
    1 X(I,J)=0.0                                                        LST02420
      DO 2 K=1,N                                                        LST02430
    2 X(K,K)=1.0                                                        LST02440
   10 DO 34 L=1,N                                                       LST02450
      KP=0                                                              LST02460
      Z=0.0                                                             LST02470
      DO 12 K=L,N                                                       LST02480
      IF(Z-DABS(A(K,L)))11,12,12                                        LST02490
   11 Z=DABS(A(K,L))                                                    LST02500
      KP=K                                                              LST02510
   12 CONTINUE                                                          LST02520
      IF(L-KP)13,20,20                                                  LST02530
   13 DO 14 J=L,N                                                       LST02540
      Z=A(L,J)                                                          LST02550
      A(L,J)=A(KP,J)                                                    LST02560
   14 A(KP,J)=Z                                                         LST02570
      DO 15 J=1,N                                                       LST02580
      Z=X(L,J)                                                          LST02590
      X(L,J)=X(KP,J)                                                    LST02600
   15 X(KP,J)=Z                                                         LST02610
   20 IF(DABS(A(L,L))-EP)50,50,30                                       LST02620
   30 IF(L-N)31,34,34                                                   LST02630
   31 LP1=L+1                                                           LST02640
      DO 36 K=LP1,N                                                     LST02650
      IF(A(K,L))32,36,32                                                LST02660
   32 RATIO=A(K,L)/A(L,L)                                               LST02670
      DO 33 J=LP1,N                                                     LST02680
   33 A(K,J)=A(K,J)-RATIO*A(L,J)                                        LST02690
      DO 35 J=1,N                                                       LST02700
   35 X(K,J)=X(K,J)-RATIO*X(L,J)                                        LST02710
   36 CONTINUE                                                          LST02720
   34 CONTINUE                                                          LST02730
   40 DO 43 I=1,N                                                       LST02740
      II=N+1-I                                                          LST02750
      DO 43 J=1,N                                                       LST02760
      S=0.0                                                             LST02770
      IF(II-N)41,43,43                                                  LST02780
   41 IIP1=II+1                                                         LST02790
      DO 42 K=IIP1,N                                                    LST02800
   42 S=S+A(II,K)*X(K,J)                                                LST02810
   43 X(II,J)=(X(II,J)-S)/A(II,II)                                      LST02820
      KER=1                                                             LST02830
      RETURN                                                            LST02840
   50 KER=2                                                             LST02850
      RETURN                                                            LST02860
      END                                                               LST02870
```

```
      REAL FUNCTION HG*8(AMPL,G,XPN,X,DEL)                                  LST00010
      IMPLICIT REAL*8 (A-H, O-Z)                                            LST00020
      DATA CR/0.017453/                                                     LST00030
      TNT = 2.0*XPN-2.0                                                     LST00040
CP    WRITE(6,991) AMPL,G,XPN,X                                             LST00050
  991 FORMAT(1X,4F10.6)                                                     LST00060
      GS = G**2                                                             LST00070
      TG = 2.0*G                                                            LST00080
      OPGS = 1.0+GS                                                         LST00090
      HG = AMPL*(1.0/((1.0/((1.0-G)**(TNT)))-(1.0/((1.0+G)**(TNT)))))      LST00100
     1 *(1.0/(OPGS-TG*DCOS(X*CR))**(XPN-1.0)                                LST00110
     2 -1.0/(OPGS-TG*DCOS((X+DEL)*CR))**(XPN-1.0))                          LST00120
CP    WRITE(6,999) AMPL,G,HG,X                                              LST00130
  999 FORMAT(1X,2F10.5,E12.4,F10.4)                                         LST00140
      IF(INUM.GE.NCF) RETURN                                                LST00150
      END                                                                   LST00160
      REAL FUNCTION EQN*8(A,X,XB,XC,NCF)                                    LST00170
      IMPLICIT REAL*8 (A-H,O-Z)                                             LST00180
      DIMENSION A(10)                                                       LST00190
      GO TO (5,55,155,255), NOF                                             LST00200
    5 DEL = A(10)                                                           LST00210
      EQN = HG(A(1),A(2),1.5,X,DEL)                                         LST00220
      RETURN                                                                LST00230
   55 DEL = A(10)                                                           LST00240
      EQN = HG(A(9),A(1),A(2),X,DEL)                                        LST00250
      RETURN                                                                LST00260
  155 DEL = XB-X                                                            LST00270
      EQN = HG(A(9),A(1),A(2),X,DEL)                                        LST00280
      RETURN                                                                LST00290
  255 EQN = A(1)*X**2 + A(2)*X + A(3)                                       LST00300
      RETURN                                                                LST00310
      END                                                                   LST00320
```

```
      SUBROUTINE PERR(ERRNO,IXS,PARAM,XS,YS,ZS)                             PER00010
      INTEGER*4 ERRNO                                                       PER00020
      DIMENSION XS(4), YS(4), ZS(4)                                         PER00030
      NUP = 4                                                               PER00040
      IF(IXS.LT.4) NUP = IXS                                                PER00050
      WRITE(6,100) ERRNO,PARAM,IXS                                          PER00060
  100 FORMAT(' ERROR DETECTED, LOCATION NO. ',I5,' PARAM = ',               PER00070
     * E14.8,' IXS = ',I10)                                                 PER00080
      WRITE(6,110) (XS(I), YS(I), ZS(I), I = 1,NUP)                         PER00090
  110 FORMAT(' COORD. FROM PERR: ',12F9.5)                                  PER00100
      RETURN                                                                PER00110
      END                                                                   PER00120
```

LIST OF REFERENCES

1. Naval Research Laboratory Report 6152, Experimental Observations of Forward Scattering of Light in the Lower Atmosphere, by J.A. Curcio and L.F. Drummeter, Jr., 30 September 1964.

2. Battelle Memorial Institute Report BAT-171-4, Feasibility of Non-Line-of-Sight Laser Communications, by G.T. Ruck, 15 December 1964.

3. King, M. and Kainer, S., "Some Parameters of a Laser-Type Beyond-the-Horizon Communications Link," Proceedings of the IEEE, v. 53, p. 137-141, February 1965.

4. Naval Electronics Laboratory Center Report 1988 (TR 1988), Extended Line-of-Sight Optical Communications Study, by G.C. Mooradian, V.J. Adrian, P.H. Levine, and W.R. Stone, p. 9, 30 June 1976, changed 4 November 1976.

5. Air Force Cambridge Research Laboratories Report 75-0255, Atmospheric Transmittance from 0.25 to 28.5 μm: Computer Code LOWTRAN 3, by J.E.A. Selby and R.A. McClatchey, p. 38-44, 7 May 1975.

6. Air Force Cambridge Research Laboratories Report 72-0497, Optical Properties of the Atmosphere, (3rd ed.), by R.A. McClatchey, R.W. Fenn, J.E.A. Selby, F.E. Volz, and J.S. Garing, p. 36-37, 69-71, 24 August 1972.

7. Aeronautical Research Associates of Princeton Report 274, Vol. I, Voice Communication via Scattered Ultraviolet Radiation, by E.S. Fishburne, M.E. Neer, and G. Sandri, February 1976.

8. Aeronautical Research Associates of Princeton Report 285, Voice Communication via Atmospheric Scattering of Ultraviolet Radiation, by E.S. Fishburne and M.E. Neer, July 1976.

9. Bhaumik, M.L., "High Efficiency UV Lasers," Laser Focus, v. 12, p. 55, February 1976.

10. Bhaumik, M.L., Bradford, R.S., Jr., and Ault, E.R., "High Efficiency KrF Eximer Laser," Applied Physics Letters, v. 28, p. 23, 1976.

11. Naval Research Laboratory Memorandum Report 3238, Ultraviolet Extinction Measurements of the Chesapeake Bay, by J.A. Curcio and G.L. Knestrick, p. 12, March 1976.

12. Shettle, E.P. and Green, A.E.S., "Multiple Scattering Calculation of the Middle Ultraviolet Reaching the Ground," Applied Optics, v. 13, p. 1567-1581, July 1974.

13. Stotts, L.B., "The Radiance Produced by Laser Radiation Transversing a Particulate Multiple-Scattering Medium," Journal of the Optical Society of America, v. 67, n. 6, p. 815-819, June 1977.

14. Bucher, E.A., "Computer Simulation of Light Pulse Propagation for Communication Through Thick Clouds," Applied Optics, v. 12, n. 10, p. 2391-2400, October 1973.

15. Bucher, E.A., "Experimenets on Light Pulse Communication and Propagation Through Atmospheric Clouds," Applied Optics, v. 12, n. 10, p. 2401-2414, October 1973.

16. Kennedy, R.S., "Communication Through Optical Scattering Channels: An Introduction," Proceedings of the IEEE, v. 58, n. 10, p. 1651-1665, October 1970.

17. Kennedy, E.A., personal communication.

18. Yarif, A., Introduction to Optical Electronics, 2d. ed., Holt, Rinehart and Winston, p. 306-307, 1976.

19. Ault, E.R., Bradford, R.S., Jr., and Bhaumik, M.L., "High-Power Xenon Flouride Laser," Applied Physics Letters, v. 27, p. 413, 1975.

20. Laser Focus, v. 13, p. 32-33, September 1977.

21. Naval Research Laboratory Report 4031, Horizontal Attenuation of Ultraviolet and Visible Light by the Lower Atmosphere, by L. Dunkelman, p. 7-10, 13, 17, 19, and 21, 10 September 1952.

22. Penndorf, R., "Tables of the Refractive Index for Standard Air and the Rayleigh Scattering Coefficient for the Spectral Region Between 0.2 and 20.0 μ and Their Application to Atmospheric Optics," Journal of the Optical Society of America, v. 47, n. 2, p. 179, February 1957.

23. Aeronautical Research Associates of Princeton Report 314, Ultraviolet Single Scattering Phase Function of Naturally Occuring Atmospheric Aerosols, by M.E. Neer, J.M. Schlupf, and P.C. Petersen, September 1977.

24. Zachor, A.S., "Aureole Radiance Field About a Source in a Scattering-Absorbing Medium," submitted for publication.

25. Pratt, W.K., Laser Communication Systems, p. 177-178, John Wiley & Sons, Inc., 1969.

26. Ross, M., Laser Receivers, John Wiley & Sons, Inc., 1966.

27. Gagliardi, R.M. and Karp, S., Optical Communications, John Wiley & Sons, 1976.

28. Valley, S.L., Handbook of Geophysics and Space Environments, Air Force Cambridge Research Laboratories, Office of Aerospace Research, U.S. Air Force, p. 7-6, 1965. (Also published by McGraw-Hill Book Co., 1965)

29. Uros, N.M., personal communication.

30. RCA, Electro-Optics Handbook, p. 114-115, 1974.

31. Selby, S.M., Standard Mathematical Tables, 23d ed., p. 103-104, CRC Press, Inc., 1975.

32. Scripps Institution of Oceanography, Reference 71-1, Underwater Lighting by Submerged Lasers and Incandescent Sources, by S.Q, Duntley, June 1971.

33. Duntley, S.Q., Journal of the Optical Society of America, V. 53, p. 214, 1963.

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